Errata for Guedj–Zeriahi, *Degenerate Complex Monge–Ampère Equations*

Corrected formulas, hypotheses, references, and proof clarifications

Independent and unofficial reader's errata

Revised 23 August 2026

Contents

Scope, edition, and conventions

Edition checked. Vincent Guedj and Ahmed Zeriahi, *Degenerate Complex Monge–Ampère Equations*, EMS Tracts in Mathematics 26 (2017), DOI 10.4171/167, ISBN 978-3-03719-167-5. This is an independent reader's errata, not an official corrigendum. This edition rechecks every candidate in the earlier draft and omits candidates that are correct as printed, merely stylistic, standard implicit arguments, or unsupported by the printed page.

Page references. Every locator uses the page number printed in the book. For the supplied PDF, an Arabic-numbered book page (p) is PDF page (p+25); Introduction page VI, for example, is PDF page 7. Statement, equation, step, paragraph, or bullet locators are added wherever the page alone would leave doubt.

What is included. The first group in each chapter records mathematical errors, missing hypotheses, notation errors that change meaning, and genuine proof clarifications. The second group records typographical and editorial corrections, including the additional items found in the second sweep; these items are integrated into the relevant chapter rather than collected in a detached appendix. Full entries use the fields **Type**, **Precise location**, **Printed text / issue**, and **Correction / repair**. Obvious spelling corrections are recorded more compactly but retain a page-and-context locator.

Verification standard. The draft supplied with the book was treated as a candidate list, not as an authority. Every retained location was rechecked against the supplied PDF, and formula-heavy or extraction-sensitive passages were checked on rendered page images. A second independent sweep was then run in four chapter-range batches with the Rethlas workflow (commit `60ca1e6`) using `gpt-5.6-sol` at `xhigh` reasoning effort; its candidates were again adjudicated against the printed pages. Thus OCR/extraction artefacts, stylistic preferences, and mathematically correct shorthand have been removed. Gaps in item identifiers record merged or rejected candidates, not unchecked pages. No independent errata can prove absolute exhaustiveness, but the list below is substantially fuller and more explicit than the supplied draft.

Front matter / Introduction

F-M01. Typo: "fourty"

- **Precise location:** Introduction, printed roman p. vi; the exact printed wording or display is reproduced or quoted under **Correction / repair**.
- **Correction / repair:** "fourty years ago" ⟶ "forty years ago".

F-M02. Typo: *compacity*

- **Precise location:** Introduction, printed roman p. x, table of contents, Chapter 14 description, phrase *Regularity and compacity properties*.
- **Correction / repair:** replace *compacity properties* by *compactness properties*.

Chapter 1. Subharmonic and plurisubharmonic functions

Range checked: printed pp. 3–38.

Mathematical corrections and proof clarifications

C1-M01. Proposition 1.1: wrong Taylor coefficient for the Laplacian

- **Precise location:** printed pp. 4–5, Proposition 1.1, Taylor expansion of the circular mean; and printed p. 13, proof of Proposition 1.19, smooth-case Laplacian limit.
- **Issue:** The proof writes the circular average expansion with coefficient $r^2\Delta h(a)/2$, and hence

$$\Delta h(a) = \lim_{r\to 0^+} \frac{2}{r^2}\left(\int_0^{2\pi} h(a+re^{i\theta})\frac{d\theta}{2\pi} - h(a)\right).$$

  With the book's normalization $\Delta = \partial_x^2 + \partial_y^2 = 4\partial_z\partial_{\bar z}$, the circular average has coefficient $r^2\Delta h(a)/4$.

- **Correction / repair:** Replace $r^2/2$ by $r^2/4$, and $2/r^2$ by $4/r^2$, at both locations. The conclusions are unaffected.

C1-M02. Proposition 1.6: sign error in the holomorphic expression for the Poisson kernel

- **Precise location:** printed p. 7, Proposition 1.6; the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** The proof writes
$$P(\zeta, z) = \frac{1-|z|^2}{|z-\zeta|^2} = \Re\left(\frac{\zeta+z}{z-\zeta}\right).$$
At $z = 0$, this gives -1, whereas the Poisson kernel is 1.
- **Correction / repair:** Replace the denominator by $\zeta - z$:
$$P(\zeta, z) = \Re\left(\frac{\zeta+z}{\zeta-z}\right).$$

C1-M03. Proposition 1.6: wrong domain symbol in the final sentence

- **Precise location:** printed p. 7, Proposition 1.6; the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** The final sentence says that h_ϕ is the unique solution to $\mathrm{DirMA}(\Delta, \phi)$. The domain under discussion is the unit disc $\mathbb{D}$; Δ denotes the Laplacian, not the domain.
- **Correction / repair:** Replace $\mathrm{DirMA}(\Delta, \phi)$ by $\mathrm{DirMA}(\mathbb{D}, \phi)$.

C1-M04. Propositions 1.8(1) and 1.28(1): composition with convex increasing functions is not literally well-posed at $-\infty$

- **Precise location:** printed p. 9; printed p. 19; the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** The statements allow subharmonic/plurisubharmonic functions taking the value $-\infty$, but assume the convex increasing function $\chi : I \to \mathbb{R}$ is defined on an interval I containing the image $u(\Omega)$. A usual real interval does not contain $-\infty$.
- **Correction / repair:** if u may attain $-\infty$, require I to be unbounded below and set $\chi(-\infty) := \lim_{t\to-\infty} \chi(t) = \inf_{t\in I} \chi(t)$. Alternatively, assume that u is finite-valued.

C1-M05. Proposition 1.13: continuity of circular means is stated but proof is too terse

- **Precise location:** printed pp. 10–11, Proposition 1.13; the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** The proof establishes monotonicity of $r \mapsto M(a, r)$, but it does not justify the asserted continuity or the limit $M(a, r) \to u(a)$.
- **Correction / repair:** Invoke the Poisson–Jensen/Riesz formula. For $0 < r < \mathrm{dist}(a, \partial\Omega)$, it writes the increment of $M(a, r)$ as an integral of the Riesz mass of concentric discs; this is continuous in $r > 0$. The submean inequality and upper semicontinuity give $u(a) \le M(a, r)$ and $\limsup_{r\downarrow 0} M(a, r) \le u(a)$, hence $M(a, r) \to u(a)$ (with the extended-value interpretation when $u(a) = -\infty$).

C1-M09. Proposition 1.30: boundary condition in the gluing construction is not literally meaningful

- **Precise location:** printed p. 20, Proposition 1.30; the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** The condition $u \geq v$ on $\partial\Omega'$ is not literal if v is only defined on Ω'. The statement uses a boundary value of v without specifying the limiting interpretation.
- **Correction / repair:** Use the limiting boundary condition

$$\limsup_{\Omega' \ni y \to x} v(y) \leq u(x), \qquad x \in \Omega \cap \partial\Omega'$$

Retain the printed compact-containment hypothesis $\Omega' \Subset \Omega$; it is harmless here and is part of the stated local gluing result.

C1-M10. Proposition 1.31(ii): denominator requires positivity of $\rho(r)$

- **Precise location:** printed p. 20, Proposition 1.31(ii); the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** The formula divides by $\rho(r)$, but the hypotheses allow $\rho(r) = 0$, for example if $\rho \equiv 0$ on $[0, r]$.
- **Correction / repair:** for the displayed fixed radius, require only $\rho(r) > 0$. If the normalized mean is to be defined for every $0 < r < \delta(a)$, assume $\rho(r) > 0$ throughout that interval.

C1-M11. Proposition 1.34 proof: radius choice in closedness of the integrability locus

- **Precise location:** printed pp. 21–22, Proposition 1.34 proof; the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** From $B(b, r) \Subset \Omega$ and $a' \in B(b, r)$, it does not follow that $B(a', r) \Subset \Omega$. The proof uses this implication when proving that the integrability locus is closed.
- **Correction / repair:** Fix $R > 0$ with $B(b, R) \Subset \Omega$, then restrict the closedness argument to $a, a' \in B(b, R/2)$ and use balls of radius at most $R/2$. Thus $B(a', R/2) \subset B(b, R) \Subset \Omega$, which supplies the compact containment actually needed in the integral estimate.

C1-M12. Proposition 1.36: continuity is false for a general right-continuous Stieltjes weight

- **Precise location:** printed p. 22, Proposition 1.36; the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** The proposition claims that the normalized Stieltjes average is continuous for any non-decreasing right-continuous γ. If γ has atoms, the normalized average generally has jumps.
- **Counterexample:** In one variable, take $u(z) = |z|^2$, $a = 0$, and $\gamma(s) = s + \mathbf{1}_{\{s \geq r_0\}}$. The atom at r_0 changes the normalized average discontinuously.
- **Correction / repair:** Replace "continuous" by "right-continuous" under the stated hypotheses, or assume γ is continuous. Also require positive denominators as in C1-M10.

C1-M14. Proposition 1.40: final convolution step appears circular / too compressed

- **Precise location:** printed p. 24, Proposition 1.40; the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** The proof writes $u * \chi_\varepsilon \to u$ in L^1_{loc}. But $u = \sup_i u_i$ is the raw envelope whose measurability and equality a.e. with u^* are part of the conclusion.
- **Correction / repair:** First apply Choquet's lemma to choose a countable subfamily whose increasing finite maxima v_j satisfy $(\lim_j v_j)^* = u^*$. Monotone convergence in L^1_{loc} gives $v := \lim_j v_j = u^*$ almost everywhere.

Since $v \le u \le u^*$, the raw envelope is measurable and equals u^* almost everywhere. Only then may one use convolution convergence.

C1-M16. Proposition 1.44: pullback may be identically $-\infty$

- **Precise location:** printed p. 26, Proposition 1.44; the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** The statement says that if $u \in \mathrm{PSH}(\Omega)$ and $f : \Omega' \to \Omega$ is holomorphic, then $u \circ f \in \mathrm{PSH}(\Omega')$. Under the book's convention PSH excludes the identically $-\infty$ function, but $f(\Omega')$ may lie in the polar set of u.
- **Correction / repair:** State that $u \circ f$ is plurisubharmonic or identically $-\infty$ on each connected component of Ω'; equivalently, add the hypothesis that $u \circ f \not\equiv -\infty$ on every component.

C1-M17. Proposition 1.44 proof: wrong source dimension

- **Precise location:** printed p. 26, proof of Proposition 1.44, sentence immediately after the second chain-rule display, beginning *Thus for all ξ*.
- **Issue:** the proof writes $\xi \in \mathbb{C}^n$, but ξ is a tangent vector in the source $\Omega' \subset \mathbb{C}^m$; the subsequent definition $\eta_k = \sum_{i=1}^m \xi_i(\partial f_k/\partial z_i)(z_0)$ confirms the source dimension.
- **Correction / repair:** replace $\xi \in \mathbb{C}^n$ by $\xi \in \mathbb{C}^m$.

C1-M18. Theorem 1.46: final epsilon step in Hartogs lemma is omitted

- **Precise location:** printed p. 29, Theorem 1.46; the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** The proof bounds $\max_K(u_j - h)$ by a quantity involving $u * \rho_\varepsilon$, but still needs to let $\varepsilon \to 0$.
- **Correction / repair:** Add that $u*\rho_\varepsilon$ decreases to u, and since h is continuous on compact K, $\max_K(u*\rho_\varepsilon - h)$ decreases to $\max_K(u - h)$.

C1-M19. Theorem 1.48, Step 2: truncation estimate has a false equality

- **Precise location:** printed p. 32, Theorem 1.48, Step 2; the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** The proof writes an equality of the form

  $$\int_K |u_j - u_j^m|^p = 2\int_{K\cap\{u_j \le -m\}} |u_j|^p,$$

  which is not correct. On $\{u_j \le -m\}$, one has $|u_j - u_j^m| = |u_j + m|$, not $2|u_j|$.
- **Correction / repair:** Replace the equality by an inequality, e.g.

  $$\int_K |u_j - u_j^m|^p \le \int_{K\cap\{u_j \le -m\}} |u_j|^p \le m^{-1}\int_K |u_j|^{p+1}.$$

C1-M20. Exercise 1.9(3): L^1 should be local

- **Precise location:** printed p. 35, Exercise 1.9(3); the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** The exercise says approximation "in L^1" on $\mathbb{C}$, but global $L^1(\mathbb{C})$ is not the natural topology and need not be finite for logarithmic-growth functions.
- **Correction / repair:** Replace L^1 by L^1_{loc}.

C1-M22. Exercise 1.20(4): connected compact sets include polar cases

- **Precise location:** printed p. 36, Exercise 1.20(4); the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** This fails already when K is a singleton.
- **Correction / repair:** Replace "a connected compact set" by "a connected compact set containing at least two points" (a non-degenerate continuum). The singleton case is polar and does not have the asserted regularity.

C1-M23. Proposition 1.25 cites Proposition 1.6 with the wrong type

- **Precise location:** printed p. 17, Proposition 1.25, paragraph immediately after the Poisson–Jensen display, sentence beginning *When u is harmonic in $\mathbb{D}$.*
- **Printed text:** *we recover the Poisson formula (Theorem 1.6).*
- **Correction / repair:** replace *Theorem 1.6* by *Proposition 1.6.*

C1-N01. Remark 1.32: the polar-coordinate formula mixes the unit and radius-r balls

- **Type:** formula error.
- **Precise location:** printed p. 21, Remark 1.32, first displayed equality after *Using polar coordinates*; in the supplied PDF this is PDF p. 46.
- **Printed text / issue:** the left-hand side integrates over the unit ball in the variable ζ, while the radial integral on the right stops at r. These are incompatible changes of variables.
- **Correction / repair:** if $d\sigma$ is normalized surface measure on S^{2n-1}, write

$$\int_{B_1} u(a + r\zeta)\, dV(\zeta) = 2n\kappa_{2n} \int_0^1 s^{2n-1} \left(\int_{S^{2n-1}} u(a + rs\xi)\, d\sigma(\xi) \right) ds.$$

  Equivalently, keep the upper limit r only after replacing the left-hand side by an integral over $B(a, r)$ and inserting the corresponding Jacobian normalization.

C1-N02. Proposition 1.39: the compact family used in the sequential proof is not defined

- **Type:** proof clarification.
- **Precise location:** printed p. 23, proof of Proposition 1.39, last paragraph, clause *since $\mathcal{U}$ is compact hence bounded*; PDF p. 48.
- **Printed text / issue:** the proof starts with an arbitrary sequence $u_j \to u$ but then invokes a compact set $\mathcal{U}$ that has not been introduced.
- **Correction / repair:** define $\mathcal{U}' = \{u, u_1, u_2, \ldots\}$. A convergent sequence together with its limit is compact in the local L^1 topology, so the required local bound applies to $\mathcal{U}'$. Equivalently, derive the same bound directly from $u_j \to u$ in L^1_{loc}.

C1-N03. Proposition 1.28: wrong variable in the parameter integral

- **Type:** notation error.
- **Precise location:** printed p. 19, Proposition 1.28, parameter-integral formula containing $E(z; t)\, d\mu(x)$.
- **Correction / repair:** replace $E(z; t)\, d\mu(x)$ by $E(z; x)\, d\mu(x)$, so that the integrand uses the variable of integration.

C1-N04. Exercise 1.2: cross-reference has the wrong type

- **Type:** cross-reference error.
- **Precise location:** printed p. 32, Exercise 1.2, reference to "(1.37)" in the instruction.
- **Correction / repair:** replace "(1.37)" by "Corollary 1.37."

Typographical and editorial corrections

- printed p. 3: "domain.Recall" → "domain. Recall".
- printed p. 9: "conseqence" → "consequence".
- printed p. 14: "writen" → "written".
- printed p. 19: "satifies" → "satisfies".
- printed p. 21: "Fix $u \in \mathrm{PSH}(\Omega)$ et let G" → "Fix $u \in \mathrm{PSH}(\Omega)$ and let G".
- printed p. 24: "differerential inequalities" → "differential inequalities".
- printed p. 28: "substracting a constant" → "subtracting a constant".
- printed p. 29: "the fist term" → "the first term".
- printed p. 35: "uncoutable" → "uncountable".

Chapter 2. Positive currents and Lelong numbers

Range checked: printed pp. 39–74.

Mathematical corrections and proof clarifications

C2-M01a. Proposition 2.18: coefficient indices use the wrong length

- **Precise location:** printed p. 47, Proposition 2.18, first local coefficient expansion after the words *If we set* $q = n - p$; the affected display is reproduced below.
- **Printed text / issue:** a current declared to have bidegree (p, p) is expanded using multi-indices of length $q = n - p$.
- **Correction / repair:** use $|I| = |J| = p$. A bidegree (p, p) current acts on test forms of bidegree $(n-p, n-p)$.

C2-M01b. Integration currents: degree and bidimension are interchanged

- **Precise location:** printed p. 50, paragraph beginning *More generally, if Z is an analytic subset*, and Theorem 2.22 immediately below it.
- **Issue:** if $\dim_{\mathbb{C}} Z = m$, the paragraph first applies $[Z]$ to an $(n - m, n - m)$-form, while Theorem 2.22 calls $[Z]$ a current of bidegree (m, m).
- **Correction / repair:** $[Z]$ acts on (m, m)-test forms, has bidegree $(n - m, n - m)$, and has bidimension (m, m).

C2-M01c. Definition 2.23: the subsequent local expansion again uses bidimension indices

- **Precise location:** printed p. 51, first display after Definition 2.23, beginning *Locally we can write*.
- **Issue:** the current is declared to have bidegree (p, p), but its coefficient indices are again assigned length $q = n - p$.
- **Correction / repair:** replace the index condition by $|I| = |J| = p$.

C2-M02. Lelong-number normalization: the constant κ_{2n} is incompatible with formula (2.3.1)

- **Precise location:** printed p. 52, Section 2.3.1, the definitions of the spherical and ball means and formula (2.3.1); the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** The definition of κ_{2n} and formula (2.3.1) are inconsistent. The Euclidean volume constant should satisfy
$$\kappa_{2n} = \frac{\sigma_{2n-1}}{2n},$$
rather than $2n\sigma_{2n-1}$. The spherical-mean formula for V_u has to be normalized accordingly.
- **Correction / repair:** Replace $\kappa_{2n} = 2n\,\sigma_{2n-1}$ by $\kappa_{2n} = \sigma_{2n-1}/(2n)$, the volume of the Euclidean unit ball when σ_{2n-1} denotes the area of the unit sphere, and use this value in (2.3.1).

C2-M03. Theorem 2.37: the final formula has an extra factor of σ

- **Precise location:** printed p. 58, proof of Theorem 2.37; the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** In the special case where ψ is constant, the formula should give
$$\nu_u(0) = -\psi^*(0),$$
not $-\psi^*(0)/\sigma$. The printed expression therefore has an extra division by σ.
- **Correction / repair:** Delete the final factor $1/\sigma$; the special case must read $\nu_u(0) = -\psi^*(0)$.

C2-M07. Theorem 2.48: a logarithm in the Fubini–Tonelli expression has the wrong sign

- **Precise location:** printed p. 68, proof of Theorem 2.48; the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** The proof contains the factor $\log(|z-a|/R)$ in a place where the integrand should be non-negative. The intended factor is
$$\log \frac{R}{|z-a|}.$$
- **Correction / repair:** Replace $\log(|z-a|/R)$ by $\log(R/|z-a|)$ in the Fubini–Tonelli calculation; the final sign then follows by the stated rearrangement.

C2-M09. Exercise 2.5: compact support does not force a positive current to vanish

- **Precise location:** printed p. 72, Exercise 2.5, complete statement.
- **Issue:** as printed, a compactly supported positive top-degree current is simply a compactly supported positive measure and need not vanish. Merely adding *closed* does not repair the top-degree case, because every top-degree current is closed.
- **Correction / repair:** assume that T is a closed positive current of bidegree (p,p) with $p < n$ and compact support. Equivalently, formulate the exercise for a closed positive current of positive bidimension.
- **Proof of the repaired statement:** choose a cutoff $\chi \equiv 1$ on a neighborhood of $\operatorname{supp} T$ and put $\beta = dd^c|z|^2$. Since $p < n$, pair T with $\beta^{,n-p}$ and integrate by parts: $\langle T, \chi\beta^{,n-p}\rangle = \langle T, dd^c(\chi|z|^2\beta^{,n-p-1})\rangle = 0$. Positivity forces this trace mass to vanish, hence $T = 0$.

C2-M10. Exercise 2.9 has the wrong multi-index dimension and repeats f_1

- **Precise location:** printed p. 73, Exercise 2.9; the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** The exercise takes $f = (f_1, \ldots, f_k)$ but writes $\alpha \in \mathbb{R}^n_+$. The displayed expression also repeats f_1 where the full list $f_1, \ldots, f_k$ is intended.

- **Correction / repair:** Replace $\alpha \in \mathbb{R}^n_+$ by $\alpha \in \mathbb{R}^k_+$, and replace the repeated occurrences of f_1 by $f_1, \dots, f_k$ in the indicated multi-index expression.

C2-N01. Corollary 2.12: the final bidegree is complementary to the stated one

- **Type:** bidegree error.
- **Precise location:** printed p. 45, Corollary 2.12, final sentence describing the current obtained from a (p,p)-form.
- **Correction / repair:** the current acts on test forms of bidegree (p,p) and therefore has bidegree $(n-p, n-p)$ (equivalently, bidimension (p,p)).

C2-N02. Integration currents: multiplicities must be specified consistently

- **Type:** convention clarification.
- **Precise location:** printed pp. 50, 57, and 72: the paragraph defining $[Z]$, the Poincaré–Lelong formula for a divisor, and the related exercise.
- **Printed text / issue:** the notation alternates between the reduced analytic set and a divisor with multiplicities without saying which cycle is being integrated.
- **Correction / repair:** state once that $[D] = \sum_j m_j [D_j]$ for the divisor $D = \sum_j m_j D_j$; if only the reduced support is intended, write $[|D|] = \sum_j [D_j]$.

C2-N03. Example 2.39: the local expansions omit Taylor remainders

- **Type:** formula clarification.
- **Precise location:** printed p. 59, Example 2.39, the two local expansions used to compare vanishing orders.
- **Printed text / issue:** finite leading terms are written as exact equalities although higher-order terms are present in general.
- **Correction / repair:** append the appropriate $O(|z|^{m+1})$ (respectively higher-order) remainder, or explicitly say that only the initial homogeneous term is being displayed.

C2-N04. Theorem 2.44: the Schur-complement step must minimize over the auxiliary variable

- **Type:** omitted proof step.
- **Precise location:** printed p. 63, proof of Theorem 2.44, block-matrix positivity argument containing the auxiliary vector t.
- **Correction / repair:** minimize the associated quadratic form in t. Substitution of the minimizing value gives the Schur complement and the inequality used in the next line; positivity for one arbitrary t is not enough.

C2-N05. Theorem 2.45: wrong cross-reference and an implicit finite-generation step

- **Type:** cross-reference and proof clarification.
- **Precise location:** printed p. 64, proof of Theorem 2.45, sentence citing Corollary 2.36 and the subsequent choice of finitely many local generators.
- **Correction / repair:** cite Corollary 2.42. The passage to finitely many generators uses the Noetherian property of the local holomorphic ring after shrinking the neighborhood; state this explicitly.

C2-N06. Recalled Lelong-mass formula has the wrong dimensional normalization

- **Type:** normalization error.
- **Precise location:** printed p. 64, recalled formula for the mass ratio of a positive closed $(1,1)$-current over $B(a,r)$.
- **Correction / repair:** use the denominator $\kappa_{2n-2}r^{2n-2}$, matching the degree of the transverse volume form.

C2-N07. Lemma 2.46: an equality of forms only holds after restriction to the sphere

- **Type:** domain-of-identity clarification.
- **Precise location:** printed p. 65, proof of Lemma 2.46, displayed equality involving the radial form and the standard contact form.
- **Printed text / issue:** the two ambient forms are not equal on the full punctured space.
- **Correction / repair:** pull them back to the sphere of fixed radius, or state the equality only on tangent vectors to that sphere; the radial component then vanishes.

C2-N08. Theorem 2.50: the uniform estimate requires a strict intermediate threshold

- **Type:** omitted compactness argument.
- **Precise location:** printed pp. 70–71, proof of Theorem 2.50, passage from pointwise Lelong-number bounds on a compact set K to a uniform radius δ.
- **Correction / repair:** choose λ with $\nu(K,U) < \lambda < 2$. Upper semicontinuity provides a neighborhood of each point on which the relevant Lelong number is $< \lambda$; compactness of K supplies a finite subcover and one uniform δ.

Typographical and editorial corrections

- printed p. 39 / Section 2.1.1 heading/TOC: "Forms with distributions coefficients" ⟶ "Forms with distribution coefficients".
- printed p. 51, first paragraph after Definition 2.23: "to the the basis" ⟶ "to the basis".
- printed p. 57, proof of Theorem 2.36: "in the the analytic set" ⟶ "in the analytic set".
- printed p. 59, Example 2.39: "whence the equailty" ⟶ "whence the equality".
- printed p. 65, paragraph after Lemma 2.46: "when studing the Monge–Ampère operator" ⟶ "when studying the Monge–Ampère operator".
- printed p. 66, paragraph beginning with the computation: "A straightfoward computation" ⟶ "A straightforward computation".
- printed p. 69, proof of Theorem 2.49: "Substracting a uniform constant" ⟶ "Subtracting a uniform constant".
- printed p. 70, proof of Theorem 2.50: "estimate u_3 fom below" ⟶ "estimate u_3 from below".

Chapter 3. Bedford–Taylor complex Monge–Ampère operator

Range checked: printed pp. 75–100.

Mathematical corrections and proof clarifications

C3-M01. Definition 3.2 / Proposition 3.3: $du \wedge d^c u \wedge T$ is not generally closed

- **Precise location:** printed pp. 76–77, Definition 3.2 / Proposition 3.3; the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** Positivity is correct, but the current $du \wedge d^c u \wedge T$ is not generally closed. In the smooth case,
$$d(du \wedge d^c u \wedge T) = -du \wedge dd^c u \wedge T,$$
which need not vanish.
- **Correction / repair:** Delete "closed" wherever it modifies $du \wedge d^c u \wedge T$ in Definition 3.2/Proposition 3.3. The related current $dd^c(u^2) \wedge T = 2(du \wedge d^c u + u\, dd^c u) \wedge T$ is closed, but its first summand need not be.

C3-M02. Proposition 3.6: the parameter M is unused and then redefined

- **Precise location:** printed p. 78, Proposition 3.6; the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** The statement fixes a compact set $K \subset \Omega$ and $M > 0$, but this M is not used; the proof later redefines M as a norm of u.
- **Correction / repair:** Delete the initial $M > 0$.

C3-M03. Proposition 3.6: the proof should choose $0 < b < a$, not $a < b$

- **Precise location:** printed p. 78, Proposition 3.6; the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** Since the sublevel domains $D_c = \{\rho < -c\}$ shrink as c increases, the annulus and gluing argument require $D_a \subset D_b$, hence $0 < b < a$. The printed proof says $a < b$.
- **Correction / repair:** Replace $a < b$ by $0 < b < a$.

C3-M04. Proposition 3.6: the final inequality has the wrong direction

- **Precise location:** printed p. 78, Proposition 3.6; the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** The statement/proof gives an inequality of the form $u \le \tilde{u} \le A\rho$, but the construction is $\tilde{u} = \max\{u, A\rho\}$, which is not compatible with the upper bound $\tilde{u} \le A\rho$.
- **Correction / repair:** Replace item (iii) by $A\rho \le \widetilde{u} \le 0$. This is exactly what the three-piece construction proves: on D_a, $A\rho \le -M \le u$; on $D_b \setminus D_a$ it is the defining maximum; and outside D_b equality $\widetilde{u} = A\rho$ holds. The unused global comparison $u \le \widetilde{u}$ should be deleted.

C3-M05. Proposition 3.15: the integration-by-parts step does not satisfy the stated hypotheses

- **Precise location:** printed pp. 85–86, proof of Proposition 3.15; the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** The proof uses
$$\int_D (-\phi_j) dd^c \rho \wedge T \wedge \beta_{p-1} = \int_D (-\rho) dd^c \phi_j \wedge T \wedge \beta_{p-1},$$
citing Proposition 3.7. But Proposition 3.7 requires both functions to have boundary value 0; ρ does, whereas $\phi_j = \max(\phi, -j)$ generally does not.
- **Correction / repair:** The equality cannot simply be replaced by a useful inequality: Proposition 3.7 gives
$$\int_D (-\phi_j)\, dd^c \rho \wedge T \wedge \beta^{p-1} \ge \int_D (-\rho)\, dd^c \phi_j \wedge T \wedge \beta^{p-1},$$

the opposite direction from the estimate needed in the printed proof. A corrected proof must first modify ϕ_j to have the same boundary value as ρ, or perform a cutoff integration by parts and bound the resulting boundary/collar term using $-M \leq \phi \leq 0$ near ∂D.

C3-M06. Lemma 3.22: the list $u_1, \ldots, u_n$ should be $u_1, \ldots, u_p$

- **Precise location:** printed p. 89, Lemma 3.22; the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** The current is $T = \bigwedge_{1\leq i\leq p} dd^c u_i$, so the later phrase should refer to $u_1, \ldots, u_p$, not $u_1, \ldots, u_n$.
- **Correction / repair:** Replace $u_1, \ldots, u_n$ by $u_1, \ldots, u_p$ in the indicated sentence.

C3-M08. Example 3.32 / Exercise 3.8: the printed Kiselman example is not psh on the whole ball

- **Precise location:** printed p. 95, Example 3.32, and printed pp. 98–99, Exercise 3.8; the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** Writing $z' = (z_2, \ldots, z_n)$, the Schur complement of the complex Hessian of $\varphi = (-\log|z_1|)^{1/n}(|z'|^2-1)$ has the sign of $1-1/n-|z'|^2$. Thus φ is not plurisubharmonic at points of the unit ball with $|z'|^2 > 1-1/n$.
- **Correction / repair:** Restrict the example and Exercise 3.8 to $\{|z'|^2 < 1-1/n,\ 0 < |z_1| < 1\}$; in particular, it is valid on the punctured ball $B(0, \sqrt{1-1/n}) \setminus \{z_1 = 0\}$. Alternatively, change the constant (1) in the definition and state the corresponding smaller domain explicitly.

C3-M10. Exercise 3.4: the chain-rule formula misses a binomial factor

- **Precise location:** printed p. 97, Exercise 3.4; the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** The displayed chain-rule formula lacks the binomial factor n.
- **Correction / repair:** The second term should be

$$n\,\chi''(u)\chi'(u)^{n-1} du \wedge d^c u \wedge (dd^c u)^{n-1}.$$

C3-M11. Exercise 3.5: proper push-forward misses the topological degree

- **Precise location:** printed pp. 97–98, Exercise 3.5; the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** Under a proper holomorphic map of degree d, the push-forward of the Monge–Ampère measure acquires the topological degree. The printed formula omits this factor.
- **Correction / repair:** Add the factor d, or assume explicitly that the map has degree one.

C3-M12. Exercise 3.6: the constant is inconsistent with the book's normalization

- **Precise location:** printed p. 98, Exercise 3.6; the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** With the book's convention $dd^c \log|z| = \delta_0$, one has $dd^c(\operatorname{Im} z)_+ = (2\pi)^{-1}\lambda_{\mathbb{R}}$ in one complex dimension. Hence the factors $(4\pi)^{-n}$ in part (1) and $(n!/(2\pi))^n$ in part (3) are inconsistent.
- **Correction / repair:** In part (1) replace $(4\pi)^{-n}$ by $(2\pi)^{-n}$. In part (3) replace the printed constant by $n!/(2\pi)^n$.

C3-M13. Exercise 3.10: the sequence converges to half of the intended function

- **Precise location:** printed p. 99, Exercise 3.10; the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** The sequence
$$\frac{1}{2j}\max(0, \log|z_1^j + \cdots + z_n^j|)$$
converges to one half of the expected maximum-type function.
- **Correction / repair:** Remove the factor 1/2, or change the target function accordingly.

C3-N01. Exercise 3.7: the displayed examples are only locally bounded

- **Type:** false boundedness assertion.
- **Precise location:** printed p. 98, Exercise 3.7, sentence asserting that the functions u_j and v_j are bounded.
- **Printed text / issue:** the functions grow along unbounded sequences; for example, evaluating at $z = (R, \dots, R)$ and letting $R \to \infty$ makes the claimed global bound impossible.
- **Correction / repair:** replace "bounded" by "locally bounded." This is the property needed for the local Bedford–Taylor products in the exercise.

C3-N02. Exercise 3.17: rank $n-1$ is not enough to produce a global kernel foliation when the rank jumps

- **Type:** missing constant-rank hypothesis.
- **Precise location:** printed p. 100, Exercise 3.17, assertion that the kernels of $dd^c u$ define a holomorphic foliation.
- **Printed text / issue:** for $u(z_1, z_2) = |z_1 z_2|^2$, the rank jumps and the limiting kernel lines along the two coordinate axes are incompatible at their intersection.
- **Correction / repair:** assume that $dd^c u$ has constant rank $n-1$ on the region under consideration, or restrict the conclusion to the regular constant-rank locus. There the kernel is a smooth complex line distribution; closedness of $dd^c u$ yields involutivity, so Frobenius gives the leaves, and the restriction of u to each leaf is harmonic.

Typographical and editorial corrections

- printed p. 85, proof following Proposition 3.15: "stricly plurisubharmonic" $\longrightarrow$ "strictly plurisubharmonic".
- printed p. 86, discussion of continuity: "discontinous for the the L^1_{loc}-convergence" $\longrightarrow$ "discontinuous for the L^1_{loc}-convergence".
- printed pp. 94–95, proof of Corollary 3.31, choice of the auxiliary constant: "is choosen so that" $\longrightarrow$ "is chosen so that".
- printed p. 97, proof paragraph containing the repeated verb: "u is is plurisubharmonic" $\longrightarrow$ "u is plurisubharmonic".

Chapter 4. Monge–Ampère capacity and convergence in capacity

Range checked: printed pp. 101–130.

Mathematical corrections and proof clarifications

C4-M01a. Definition 4.6: inner regularization omits $K \subset E$

- **Precise location:** printed p. 103, Definition 4.6, display defining $c(E)$ from compact sets.
- **Issue:** the supremum is taken over all compact K; without $K \subset E$, its value is independent of E.
- **Correction / repair:** take the supremum over compact $K \subset E$.

C4-M01b. Theorem 4.7: the compact-set supremum omits containment in the Borel set

- **Precise location:** printed p. 104, Theorem 4.7, first displayed equality in the theorem.
- **Issue / correction:** insert $K \subset B$ in the compact-set supremum.

C4-M01c. Corollary 4.36: the same containment is missing

- **Precise location:** printed p. 123, Corollary 4.36, displayed compact approximation of capacity.
- **Issue / correction:** insert $K \subset E$ (with K compact) in the supremum.

C4-M03. Proposition 4.18(4): the sublevel-domain comparison is ordered incorrectly

- **Precise location:** printed p. 109, proof of Proposition 4.18(4), sentence beginning *Fix a plurisubharmonic defining function* and the choice of $\Omega_c = \{\rho < -c\}$.
- **Issue:** the printed inclusion makes capacity monotonicity run in the wrong direction. Merely reversing the inclusion to $\Omega' \Subset \Omega_c \Subset \Omega''$ is not justified for sublevels of an arbitrary defining function.
- **Correction / repair:** use a local finite-cover argument. Cover $\overline{\Omega'}$ by finitely many concentric pairs $B_j \Subset B_j' \Subset \Omega''$. On each pair, the explicit strictly plurisubharmonic quadratic barrier extends admissible tests and gives
$$\mathrm{Cap}(E \cap B_j; B_j') \le A_j \ \mathrm{Cap}^*(E \cap B_j; \Omega).$$
Domain monotonicity and finite subadditivity then yield
$$\mathrm{Cap}(E; \Omega'') \le \Big(\sum_j A_j\Big) \mathrm{Cap}^*(E; \Omega).$$

C4-M08. Theorem 4.26: the wedge-product index is wrong

- **Precise location:** printed p. 114, Theorem 4.26; the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** The displayed wedge product is indexed by $1 \le j \le p$, but it should be i.
- **Correction / repair:** Replace $\bigwedge_{1\le j\le p} dd^c u_{ij}$ by $\bigwedge_{1\le i\le p} dd^c u_{ij}$ on the left-hand side; make the same index change in the limiting product.

C4-M09. Theorem 4.29: the exponent β^{n-p-q} contains an undefined p

- **Precise location:** printed p. 116, proof of Theorem 4.29; the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** The proof contains β^{n-p-q} although p is not part of the local notation at that point.
- **Correction / repair:** Replace every β^{n-p-q} in this proof by β^{n-q}, so that the wedge of q currents of bidegree $(1,1)$ has top degree.

C4-M10. Theorem 4.34: the last arbitrary-set display applies dd^c to an unregularized envelope

- **Precise location:** printed p. 121, proof of Theorem 4.34, final display for an arbitrary set E, where the Monge–Ampère operator is written using h_E.
- **Issue:** for arbitrary E, the raw envelope h_E need not be upper semicontinuous or plurisubharmonic, so $(dd^c h_E)^n$ is not the object defined by the preceding theory. The earlier uses for open G are legitimate because $h_G = h_G^*$.
- **Correction / repair:** replace only this final occurrence under dd^c by h_E^*.

C4-M11. Example 4.37 and Exercise 4.13: ball centered at a uses $\log |z|$

- **Precise location:** printed p. 123, Example 4.37, and printed p. 129, Exercise 4.13; the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** For a ball $B(a, r)$, the formula should involve $\log |z - a|$, not $\log |z|$.
- **Correction / repair:** Replace $\log |z|$ by $\log |z - a|$.

C4-M13. Corollary 4.44: the admissible function is written with the wrong variable

- **Precise location:** printed pp. 126–127, proof of Corollary 4.44; the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** The proof writes an admissibility condition involving $u \le -1$, but the construction requires $h+\varepsilon u \le -1$ on E.
- **Correction / repair:** Replace the displayed condition by the one involving $h + \varepsilon u$.

C4-N01. Corollary 4.13: the abstract countable-union step requires a countability hypothesis

- **Type:** missing topological hypothesis.
- **Precise location:** printed p. 106, proof of Corollary 4.13, passage expressing the open set as a countable union of relatively compact sets.
- **Correction / repair:** in the abstract formulation assume the space is σ-compact (second countability suffices here). The actual domain $\Omega \subset \mathbb{C}^n$ has this property, so the book's application is valid.

C4-N02. Theorem 4.20: the two approximation errors consume the same epsilon budget

- **Type:** quantitative proof gap.
- **Precise location:** printed pp. 110–111, proof of Theorem 4.20, the two consecutive approximations each bounded by ε before the final estimate.
- **Correction / repair:** choose each preliminary error $< \varepsilon/2$ (or start with $\varepsilon/2$ throughout). The triangle inequality then yields the announced final bound $< \varepsilon$.

C4-N03. Theorem 4.40: wrong numbered cross-reference

- **Type:** cross-reference error.
- **Precise location:** printed p. 124, proof of Theorem 4.40, citation "Theorem 4.34(2)."
- **Correction / repair:** replace it by "Theorem 4.32(2)." The cited assertion is the volume–capacity estimate in that theorem.

C4-N04. Exercise 4.14: the extremal function must be upper-semicontinuously regularized

- **Type:** regularization omission.
- **Precise location:** printed p. 129, Exercise 4.14, displayed assertion involving the Siciak extremal function V_E.
- **Correction / repair:** replace V_E by its upper-semicontinuous regularization V_E^* wherever plurisubharmonicity or its Monge–Ampère measure is used.

Typographical and editorial corrections

- printed p. 102, "arrange so that that the corresponding open sets" $\longrightarrow$ "arrange so that the corresponding open sets".
- printed p. 107, "Exercice 4.6" $\longrightarrow$ "Exercise 4.6".
- printed p. 112, "Let $(f_j)_{j\geq 0}$ be sequence" $\longrightarrow$ "Let $(f_j)_{j\geq 0}$ be a sequence".
- printed p. 123, "see Exercice 4.8" $\longrightarrow$ "see Exercise 4.8".
- printed p. 102, discussion before Definition 4.2: "the relation beween these two notions" $\longrightarrow$ "the relation between these two notions".
- printed p. 122, proof preceding Corollary 4.36: "by subaddivity" $\longrightarrow$ "by subadditivity".
- printed p. 124, paragraph following Theorem 4.40: "n-dimentional Lebesgue measure" $\longrightarrow$ "n-dimensional Lebesgue measure".
- printed p. 124, same discussion, sentence beginning "Lelong asked": "wether" $\longrightarrow$ "whether".

Chapter 5. The local Dirichlet problem

Range checked: printed pp. 131–160.

Mathematical corrections and proof clarifications

C5-M01. Definition 5.3: Green function formula is not defined at $y = 0$

- **Precise location:** printed p. 133, Definition 5.3; the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** The term $K_N(|y|x - y/|y|)$ is undefined when $y = 0$, and the first term is singular when $x = y$. Thus the assertion immediately below the definition that G is well defined on all of $B \times B$ is false literally.
- **Correction / repair:** Give the printed formula for $y \neq 0$ and $x \neq y$, set

  $$G(x, 0) = K_N(x) - K_N(e), \qquad |e| = 1,$$

  and set $G(y, y) = -\infty$. The value $K_N(e)$ is independent of the choice of the unit vector e, because K_N is radial.

C5-M02. Poisson kernel constant is wrong

- **Precise location:** printed p. 134; also affects Proposition 5.5, Theorem 5.6, Exercises 5.2-5.3; the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** The displayed Poisson kernel contains an extra factor $(N - 2)^{-1}$.
- **Correction / repair:** Replace it by

  $$P(x, y) = \frac{1}{\sigma_{N-1}} \frac{1 - |x|^2}{|x - y|^N}, \qquad x \in B,\ y \in \partial B.$$

  Use this normalization also in Proposition 5.5, Theorem 5.6, and Exercises 5.2–5.3.

C5-M03. Proposition 5.7: wrong codomain for u

- **Precise location:** printed p. 134, Proposition 5.7; the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** The statement says $u : \Omega \to \mathbb{R}^N$ is upper semicontinuous. A scalar subharmonic function should take values in $[-\infty, +\infty)$, not in $\mathbb{R}^N$.
- **Correction / repair:** Replace by $u : \Omega \to [-\infty, +\infty)$.

C5-M04. Proposition 5.7 proof: wrong Laplacian coefficient

- **Precise location:** printed p. 135, Proposition 5.7 proof; the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** For $h_\varepsilon = h - \varepsilon(|x-a|^2 - r^2)$ in $\mathbb{R}^N$, the real Laplacian is $\Delta h_\varepsilon = -2N\varepsilon$, not $-2n\varepsilon$.
- **Correction / repair:** Replace $-2n\varepsilon$ by $-2N\varepsilon$.

C5-M05. Lemma 5.8 proof: diagonalization matrix is misstated

- **Precise location:** printed p. 136, Lemma 5.8 proof; the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** The proof says that a positive Hermitian matrix Q can be written as $Q = PAP^{-1}$ with P an "invertible Hermitian matrix". The usual diagonalization is by a unitary matrix. Moreover the assertion $H' = PHP^{-1} \in H_n^+$ is justified if P is unitary, not for a general Hermitian invertible P.
- **Correction / repair:** Replace "invertible Hermitian" by "unitary". Then $H' = PHP^{-1} = PHP^*$ is again positive Hermitian and has the required determinant.

C5-M06. Proposition 5.9(ii): malformed quantifier over test functions

- **Precise location:** printed p. 136, Proposition 5.9(ii); the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** The statement says "all functions that are qC^2 in a neighborhood…".
- **Correction / repair:** Write: "for all $z_0 \in \Omega$ and all C^2 functions q in a neighborhood B of z_0 such that …".

C5-M08. Proposition 5.9 proof: displayed Laplacian constant is inconsistent with the notation

- **Precise location:** printed p. 137, Proposition 5.9 proof; the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** The proof again writes $\Delta q(x_0) = -2n\varepsilon$. If Δ is the real Laplacian on $\mathbb{C}^n = \mathbb{R}^{2n}$, the coefficient should be $-4n\varepsilon$; if it is the complex Laplacian $\sum \partial^2/(\partial z_j \partial \bar{z}_j)$, the coefficient should be $-n\varepsilon$. The displayed $-2n\varepsilon$ matches neither convention.
- **Correction / repair:** This paragraph invokes Proposition 5.7, whose Δ is the real Laplacian on $\mathbb{R}^{2n}$. Replace the displayed value by $-4n\varepsilon$.

C5-M09. Proposition 5.9, $(iii) \Rightarrow (i)$: wrong class of matrices in the smooth argument

- **Precise location:** printed p. 138, Proposition 5.9, $(iii)(i)$; the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** The proof says $\mathrm{tr}(HA) \geq 0$ for all $H \in H_n$. The hypothesis only gives this for positive Hermitian H with determinant n^{-n}.
- **Correction / repair:** Replace $H \in H_n$ by $H \in H_n^+$ with $\det H = n^{-n}$. This restricted family is sufficient: after diagonalizing A, vary a positive diagonal H of fixed determinant to detect any negative eigenvalue of A.

C5-M10. Definition of the Perron class V: boundary value should use upper boundary limit

- **Precise location:** printed p. 139, Definition of the Perron class V; the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** The class V is defined using $v|_{\partial\Omega} \le \varphi$, but elements of V are bounded psh functions and need not be continuous up to the boundary.
- **Correction / repair:** Replace the boundary condition by

$$\limsup_{\Omega \ni z \to \xi} v(z) \le \varphi(\xi), \qquad \xi \in \partial\Omega.$$

C5-M11. Proposition 5.11 proof: wrong upper bound in the compact subclass V_0

- **Precise location:** printed p. 140, Proposition 5.11 proof; the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** After proving all functions in V are bounded above by B, the proof defines $V_0 = v \in V | v_0 \le v \le M$. This should be B, not M; M is $||f||_{L^\infty}$ and is unrelated to the maximum-principle bound. The proof later uses $w \le B$, not $w \le M$.
- **Correction / repair:** Replace $v \le M$ by $v \le B$ in the definition of V_0.

C5-M12. Proposition 5.11 proof of stability under maxima: exceptional epsilons are misstated

- **Precise location:** printed p. 140, Proposition 5.11 proof of stability under maxima; the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** After observing that $\mu(u = v + \varepsilon) \ne 0$ for at most countably many ε, the proof says the previous case applies "for those epsilons." It should apply for all other epsilons.
- **Correction / repair:** Replace "for those ε's" by "for every ε outside this at most countable exceptional set," and take a sequence of such $\varepsilon \to 0$.

C5-M13. Theorem 5.12 Step 1: variable and grammar errors in the Choquet step

- **Precise location:** printed p. 141, Theorem 5.12 Step 1; the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** The proof says (v_j) in non-decreasing and then uses $v_j \to u$ although the envelope is denoted U.
- **Correction / repair:** Write (v_j) is non-decreasing and $v_j \to U$ in $L^1(\Omega)$.

C5-M14. Theorem 5.12 Step 2: the upper barrier estimates for w_0 are false as written

- **Precise location:** printed pp. 141–142, Theorem 5.12 Step 2; the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** With $w_0 = K\rho - \psi - 2\varepsilon$ and $|\psi - \varphi| \le \varepsilon$ on the boundary, the claimed boundary estimate $-\varphi - \varepsilon \le w_0 \le -\varphi$ is not correct; one only gets a bound with a larger epsilon loss, e.g. $-\varphi - 3\varepsilon \le w_0 \le -\varphi - \varepsilon$ on $\partial\Omega$.
- **Correction / repair:** Use exactly

$$-\varphi - 3\varepsilon \le w_0 \le -\varphi - \varepsilon \quad \text{on } \partial\Omega.$$

  The upper bound still gives $v + w_0 \le 0$ on the boundary for every $v \in V$, while the lower bound tends to $-\varphi$ as $\varepsilon \downarrow 0$.

C5-M15. Theorem 5.12 Step 3: inequality (5.2.5) misses the modulus/Lipschitz contribution of ψ

- **Precise location:** printed p. 142, Theorem 5.12 Step 3; the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** The proof claims, for $\xi \in \partial\Omega$ and $z \in \Omega$, $U(z) - \varphi(\xi) \leq -w_0(z) - \varphi(\xi) \leq -K\rho(z) + \varepsilon$. But $-w_0(z) - \varphi(\xi) = -K\rho(z) + \psi(z) + 2\varepsilon - \varphi(\xi)$, and the term $\psi(z) - \varphi(\xi)$ is not controlled by ε for arbitrary z, ξ. For the later use with z close to ξ, one needs a modulus of continuity term.
- **Correction / repair:** Replace (5.2.5) by

$$U(z) - \varphi(\xi) \leq -K\rho(z) + \omega_\psi(|z - \xi|) + 3\varepsilon,$$

  where ω_ψ is a modulus of continuity of the chosen extension ψ. Consequently, when $|z - \xi| \leq \delta$, the next line contains $KC\delta + \omega_\psi(\delta) + 3\varepsilon$, not merely $KC\delta + \varepsilon$.

C5-M16. Theorem 5.13 proof: formula (5.3.4) has the wrong order in $|a|$

- **Precise location:** printed p. 145, Theorem 5.13 proof; the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** After the second-order Taylor expansion, the displayed estimate reads $g \circ T_a + g \circ T_{-a} \leq 2g + 2C_2|a|$. The first-order terms cancel, so the error should be $O(|a|^2)$. The next displayed formula (5.3.5) uses $|a|^2$, confirming the typo.
- **Correction / repair:** Replace $2C_2|a|$ by $2C_2|a|^2$.

C5-M17. Theorem 5.13 proof: determinant factor in the pullback estimate needs absolute values and derivatives

- **Precise location:** printed p. 145, Theorem 5.13 proof; the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** The proof writes terms like $(\det T_a')^{2/n}$ and later $|\det T_a(z)|^{2/n}$. The determinant of a holomorphic Jacobian is complex, and the relevant Monge–Ampère scaling factor is $|\det T_a'(z)|^{2/n}$.
- **Correction / repair:** Use $|\det T_a'(z)|^{2/n}$ throughout. The first-order expansion should be written with the real part, e.g. $|\det T_a'(z)|^{2/n} = 1 + (2(n+1)/n)\operatorname{Re} < z, a > +O(|a|^2)$.

C5-M18. Theorem 5.13 proof: erroneous factors $1/2$ on printed p. 146

- **Precise location:** printed p. 146, Theorem 5.13 proof; the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** Since V_a was already defined as $1/2(U \circ T_a + U \circ T_{-a})$, for $v_a = V_a + C_3|a|^2(|z|^2 - 2)$ one should have $\Delta_H v_a = \Delta_H V_a + C_3|a|^2\Delta_H(|z|^2)$, not $1/2\Delta_H V_a + \ldots$. Similarly, the line $1/2V_a - C_3|a|^2 \leq 1/2V_a + \ldots \leq U$ should not contain the factor $1/2$.
- **Correction / repair:** Remove the two spurious factors $1/2$. Then the conclusion becomes $V_a \leq U + C_3|a|^2$, hence $U \circ T_a + U \circ T_{-a} - 2U \leq 2C_3|a|^2$.

C5-M19. Corollary 5.15 proof: boundary comparison line confuses boundary data and densities

- **Precise location:** printed p. 149, Corollary 5.15 proof; the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** The proof writes $Since U_k + \varepsilon(|z|^2 - 1) = f_k \leq f_j \in \partial B\ldots$. On ∂B, U_k equals the boundary datum φ_k, not the density f_k. Moreover the proof assumes φ_j increases to φ, so for $k \geq j$ the desired boundary inequality would go in the wrong direction.
- **Correction / repair:** Either approximate the boundary data with an explicit uniform error term, or insert $U_k + \varepsilon(|z|^2 - 1) - ||\varphi_k - \varphi_j||_{L^\infty(\partial B)} \leq U_j$ after applying comparison. Then let $j, k \to \infty$ and $\varepsilon \to 0$. The monotone statement for φ_j should be removed or adjusted.

C5-M21. Corollary 5.19 proof: monotonicity is reversed and the assumed subsolution is not used

- **Precise location:** printed p. 152, Corollary 5.19 proof; the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** Since $f_j = \min(f, j)$ increases to f, the corresponding solutions with fixed boundary data should decrease, not increase. The proof writes $U_j \le U_{j+1} \le u_\varphi$, and then speaks of a non-decreasing sequence. This is the wrong direction. It also never uses the assumed subsolution v, which is needed to get a uniform lower bound.
- **Correction / repair:** Use comparison to get $v \le U_{j+1} \le U_j \le u_\varphi$. Then U_j decreases to a bounded psh function U, and Bedford-Taylor monotone convergence gives $(dd^c U)^n = f\beta^n$.

C5-M22. Corollary 5.21 proof invokes the wrong theorem for L^1 densities

- **Precise location:** printed p. 152, proof of Corollary 5.21, first sentence *Applying Theorem 5.17*.
- **Issue:** Corollary 5.21 assumes $0 \le f \in L^1(B)$, but its proof says “Applying Theorem 5.17,” which only treats L^∞ densities.
- **Correction / repair:** Invoke the corrected Corollary 5.19 / Cegrell–Sadullaev subsolution theorem instead, explaining why the given local subsolution u supplies the required subsolution for the ball problem with the approximating boundary data.

C5-M23. Corollary 5.21 proof: monotonicity of U_j is again reversed

- **Precise location:** printed p. 152, Corollary 5.21 proof; the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** The proof takes φ_j decreasing to u on $\bar{B}$, but then states $u \le U_j \le U_{j+1}$. With fixed right-hand side and decreasing boundary data, comparison gives the reverse inequality $u \le U_{j+1} \le U_j$.
- **Correction / repair:** Replace by $u \le U_{j+1} \le U_j$; then take the decreasing limit.

C5-M24. Proposition 5.22 proof: inclusion of the auxiliary ball has the wrong direction

- **Precise location:** printed p. 153, Proposition 5.22 proof; the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** The proof says choose z_0 and R such that $B(z_0, R) \subset \Omega$, but the argument needs $\Omega \subset B(z_0, R)$ so that $|z - z_0|^2 - R^2 \le 0$ on Ω. The displayed final estimate also sets $R = \mathrm{diam}(\Omega)$, which is consistent with $\Omega \subset B(z_0, R)$, not with $B(z_0, R) \subset \Omega$.
- **Correction / repair:** Replace the inclusion by $\Omega \subset B(z_0, R)$, for instance with $R = \mathrm{diam}(\Omega)$ and $z_0 \in \Omega$ fixed.

C5-M25. Proposition 5.22: estimate (5.4.3) misses the exponent $1/n$

- **Precise location:** printed p. 153, Proposition 5.22; the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** Formula (5.4.2) gives a term $R^2 ||f_1 - f_2||^{1/n}$. The advertised consequence (5.4.3) is printed as $||U(\Omega, \varphi, f)|| \le R^2 ||f|| + ||\varphi||$, with no $1/n$ exponent.
- **Correction / repair:** Replace by $||U(\Omega, \varphi, f)||_{L^\infty} \le R^2 ||f||_{L^\infty}^{1/n} + ||\varphi||_{L^\infty(\partial\Omega)}$.

C5-M29. Lemma 5.25 proof: an equality should be an inequality

- **Precise location:** printed p. 155, Lemma 5.25 proof; the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** The first displayed chain contains $\int e^{w_2} f_2 \beta^n = \int (dd^c w_2)^n$, but the hypotheses only give $(dd^c w_2)^n \geq e^{w_2} f_2 \beta^n$.
- **Correction / repair:** Replace the equality by $\leq$. The proof then proceeds with the chain of inequalities.

C5-M30. Exercise 5.4: the matrix factorization is wrong

- **Precise location:** printed p. 157, Exercise 5.4; the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** The exercise says: choose unitary U with $U^* H U = D$, set $T = D^{1/2} U$, and check that T is Hermitian and $T^* T = H$. This is false in general. The displayed trace identity involving $HT^{-1}AT$ is also not correct.
- **Correction / repair:** Choose the positive Hermitian square root $T = H^{1/2}$ directly, or write $H = UDU^*$ and set $T = UD^{1/2}U^*$. Then $T = T^*$, $T^*T = TT^* = H$, and $\mathrm{tr}(T^*AT) = \mathrm{tr}(HA)$.

C5-M31. Exercise 5.6(1): the conclusion should be local/interior, not global on Ω

- **Precise location:** printed p. 157, Exercise 5.6(1); the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** The assumed second-difference estimate is only imposed for points with $dist(z, \partial\Omega) > \delta$, but the exercise asks to conclude $C^{1,1}$ and an $L^\infty(\Omega)$ bound on all of Ω.
- **Correction / repair:** Conclude the bound on the interior set $\Omega_\delta = dist(z, \partial\Omega) > \delta$ or formulate the assumption uniformly on every compact subset.

C5-M32. Exercise 5.9: measure notation is inconsistent with the class $B(\Omega, \varphi, f)$

- **Precise location:** printed p. 158, Exercise 5.9; the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** The exercise asks for a Borel measure μ, but then refers to the class $B(\Omega, \varphi, f)$, which was defined for a density $f\beta^n$, not for an arbitrary measure μ.
- **Correction / repair:** Write $B(\Omega, \varphi, \mu)$ or explicitly redefine the subsolution class with right-hand side μ.

C5-M35. Exercise 5.13: first bullet uses the wrong sequence name

- **Precise location:** printed p. 159, Exercise 5.13; the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** After asking to construct functions ψ_j, the first bullet says "the φ_j's are uniformly bounded and continuous in $\bar{B}$." The original φ_j were only the starting psh functions.
- **Correction / repair:** Replace φ_j by ψ_j in the first bullet.

C5-M36. Exercise 5.14: the stated hull formula mixes two domains

- **Precise location:** printed pp. 159–160, Exercise 5.14, opening line defining Ω, the final definition $u = U_{B,\varphi,0}$, and the displayed polynomial-hull identity.
- **Issue:** the exercise begins with a general bounded pseudoconvex Ω but later switches to the unit ball B. Moreover, the hull necessarily contains points over the interior (in particular the zero section over the base), so it cannot be confined to $\partial\Omega \times \mathbb{C}$ as the printed formula suggests.
- **Correction / repair:** a coherent minimal version is obtained by taking $\Omega = B$ from the outset and writing the hull over $\overline{B}$:

$$\widehat{F} = \{(z, w) \in \overline{B} \times \mathbb{C} : |w| \leq e^{-U_{B,\varphi,0}(z)}\}.$$

A general-domain version needs the corresponding polynomial-convexity hypotheses; merely replacing B by Ω is not sufficient.

Typographical and editorial corrections

- printed p. 136, "There exist an invertible Hermitian matrix P" $\longrightarrow$ "There exists a unitary matrix P" (see C5-M05 for the mathematical reason).
- printed p. 141, "(v_j) in non-decreasing" $\longrightarrow$ "(v_j) is non-decreasing".
- printed p. 150, "where where M is" $\longrightarrow$ "where M is".
- printed p. 156 integrates over $\mathbb{R}^n$ although the ambient space is $\mathbb{R}^N$; replace by $\mathbb{R}^N$.
- printed p. 156, "the the Riesz measure" $\longrightarrow$ "the Riesz measure".
- printed p. 156, "continous" $\longrightarrow$ "continuous".
- printed p. 157, "Show that that" $\longrightarrow$ "Show that".
- printed p. 159, "when φ in no better than $C^{1,1}$" $\longrightarrow$ "when φ is no better than $C^{1,1}$".
- printed p. 159, "probability measures μ_j in B and one of plurisubharmonic functions ψ_j" $\longrightarrow$ "and a sequence of plurisubharmonic functions ψ_j".
- printed p. 143, proof of the regularity assertion: "second-order partial derivates" $\longrightarrow$ "second-order partial derivatives".
- printed p. 152, paragraph describing extensions of the method: "It can be be generalized" $\longrightarrow$ "It can be generalized".
- printed p. 155, concluding literature paragraph: "an overwiev of these techniques" $\longrightarrow$ "an overview of these techniques".

Chapter 6. Viscosity solutions in domains

Range checked: printed pp. 161–187.

Mathematical corrections and proof clarifications

C6-M01. Definition 6.4: supersolution inequality has the wrong sign

- **Precise location:** printed p. 165, Definition 6.4; the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** Definition 6.4(2) says a viscosity supersolution satisfies $F(x_0, \psi(x_0), D\psi(x_0), D^2\psi(x_0)) \leq 0$ for lower tests. This contradicts Definition 6.1, Proposition 6.7, Proposition 6.11(2), and the later complex Monge–Ampère supersolution convention. For the equation $F = 0$ with the book's convention, subsolutions satisfy $F \leq 0$ and supersolutions satisfy $F \geq 0$.
- **Correction / repair:** Replace the final ≤ 0 in Definition 6.4(2) by ≥ 0.

C6-M02. Proposition 6.5 proof: "classical solution" should be "classical subsolution"

- **Precise location:** printed p. 165, Proposition 6.5 proof; the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** In the proof of the first item, the second direction begins "Assume now u is a classical solution." Only the subsolution inequality is being proved and used.
- **Correction / repair:** Replace by "Assume now u is a classical subsolution."

C6-M04. Definition 6.10: closed jets miss the value-convergence condition

- **Precise location:** printed p. 167, Definition 6.10; the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** The definition of the approximate/closed jets $\bar{J}^{2,+}u(x_0)$ and $\bar{J}^{2,-}u(x_0)$ only asks for $y_j \to x_0$ and $(p_j, Q_j) \to (p, Q)$. In the standard viscosity definition one also requires $u(y_j) \to u(x_0)$ (respectively $v(y_j) \to v(x_0)$). This condition is needed in Proposition 6.11 to pass the u-variable through a merely semicontinuous Hamiltonian.
- **Correction / repair:** Add $u(y_j) \to u(x_0)$ in the superjet case and $v(y_j) \to v(x_0)$ in the subjet case.

C6-M05. Proposition 6.11(2): wrong function in the closed subjet condition

- **Precise location:** printed p. 168, Proposition 6.11(2); the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** The supersolution statement uses $(p, Q) \in \bar{J}^{2,-}u(x_0)$ and $F(x_0, u(x_0), p, Q) \geq 0$, although the function under discussion is v.
- **Correction / repair:** Replace by $(p, Q) \in \bar{J}^{2,-}v(x_0)$ and $F(x_0, v(x_0), p, Q) \geq 0$.

C6-M06. Theorem 6.17: ambient vector space typo

- **Precise location:** printed p. 169, Theorem 6.17; the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** The theorem is stated on $\mathbb{R}^N$, but the concluding sequence is written $p_j \to 0 \in \mathbb{R}^n$.
- **Correction / repair:** Replace $\mathbb{R}^n$ by $\mathbb{R}^N$.

C6-M07. Sup/inf-convolution definition: the first supremum/infimum is over the wrong set

- **Precise location:** printed p. 170; repeated in Theorem 6.31, printed p. 180; the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** The formulas define $u^\varepsilon(x) = \sup_{y \in \Omega_\varepsilon}\{\cdots\}$ and $v_\varepsilon(x) = \inf_{y \in \Omega_\varepsilon}\{\cdots\}$. In the standard sup/inf convolution on Ω_ε, the first optimization should be over $y \in \Omega$; the equality with the restricted optimization over $|y - x| \leq A\varepsilon$ then follows from the oscillation bound and the definition of Ω_ε. Optimizing only over Ω_ε is too restrictive and is not equivalent to the displayed local formula.
- **Correction / repair:** Replace the first $y \in \Omega_\varepsilon$ by $y \in \Omega$ in both the sup- and inf-convolution definitions. The same correction is needed in the comparison proof.

C6-M09. Proposition 6.20 proof: wrong function name

- **Precise location:** printed p. 171, Proposition 6.20 proof; the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** The proof says q_H is an upper test for u at z_0, and ends by saying Proposition 5.9 shows u is plurisubharmonic. The function is φ.
- **Correction / repair:** Replace u by φ in both places.

C6-M10. Proposition 6.21 proof: wrong function names and point names

- **Precise location:** printed p. 172, Proposition 6.21 proof; the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** The proof first says q is an upper test for u, then later fixes $z_0 \in B$ although no such B is in force, and says $\varphi - q$ has a local maximum at x_0 while the fixed point is z_0.
- **Correction / repair:** Replace u by φ, $z_0 \in B$ by $z_0 \in \Omega$, and x_0 by z_0.

C6-M11. Proposition 6.21 proof: auxiliary linear solution is denoted by the wrong symbol

- **Precise location:** printed p. 172, Proposition 6.21 proof; the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** In the linear PDE reduction the proof says: choose a C^2 solution of $\Delta_H \varphi = f^{1/n}$ and then $u = \varphi - f$ satisfies $\Delta_H u \geq 0$. This reuses φ for an auxiliary solution and then subtracts the density f, which is nonsensical.
- **Correction / repair:** Introduce a new auxiliary function, say ρ, with $\Delta_H \rho = f^{1/n}$, and set $u = \varphi - \rho$. Then $\Delta_H u \geq 0$ in the viscosity sense.

C6-M12. Proposition 6.21 proof: final displayed inequality is missing the measure/form notation

- **Precise location:** printed p. 172, Proposition 6.21 proof; the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** The last approximation step displays $(dd^c \psi_\varepsilon)^n_{\mathrm{BT}} \geq f$, while the surrounding statement concerns measures/forms.
- **Correction / repair:** Write for instance $(dd^c \psi_\varepsilon)^n_{\mathrm{BT}} \geq f\beta^n$ (or the stronger displayed density used just before), and then let $\varepsilon \to 0$.

C6-M13. Theorem 6.22 proof: converse starts from the wrong hypothesis

- **Precise location:** printed p. 173, Theorem 6.22 proof; the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** The converse direction begins "Conversely, fix z_0 and assume (i) holds." It should assume (ii), since the goal is to prove (i).
- **Correction / repair:** Replace "assume (i) holds" by "assume (ii) holds."

C6-M14. Theorem 6.22 proof: undefined measure symbol

- **Precise location:** printed p. 173, Theorem 6.22 proof; the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** The first direction concludes $(dd^c \max\{\varphi, \rho - c\})^n_{\mathrm{BT}} \geq \nu$, but ν is not defined in the theorem.
- **Correction / repair:** Replace ν by $f\beta^n$.

C6-M17. Proposition 6.25 proof: wrong reference for the viscosity sup-convolution step

- **Precise location:** printed p. 174, Proposition 6.25 proof; the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** In the first implication, after assuming φ is a viscosity subsolution, the text says that φ^δ satisfies the corresponding viscosity inequality "see Lemma 6.26." But Lemma 6.26 is a pluripotential statement, not the viscosity sup-convolution statement.
- **Correction / repair:** Refer instead to Proposition 6.18 / Exercise 6.6, or insert the direct sup-convolution viscosity argument.

C6-M18. Proposition 6.25 proof: undefined measure symbol in the limit

- **Precise location:** printed p. 174, Proposition 6.25 proof; the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** The proof concludes $(dd^c \varphi)^n_{\mathrm{BT}} \geq e^\varphi \nu$ in the pluripotential sense. The symbol ν is not defined in the proposition.
- **Correction / repair:** Replace by $e^\varphi f\beta^n$.

C6-M19. Proposition 6.25 proof: several local variable typos in the test-function argument

- **Precise location:** printed p. 175, Proposition 6.25 proof; the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** The definition of q_ε reads $q_\varepsilon(z) := q(x) + \ldots$; later the proof claims $z_\delta \to x_0$, although the point is z_0. The proof also uses q^δ without defining it explicitly as the sup-convolution of the smooth function q.
- **Correction / repair:** Replace $q(x)$ by $q(z)$, replace x_0 by z_0, and define q^δ before using the uniform estimate $q^\delta - q \to 0$.

C6-M20. Lemma 6.26 proof: wrong envelope named in the countable-subfamily argument

- **Precise location:** printed p. 176, Lemma 6.26 proof; the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** The proof says "Since φ is the upper envelope of the family $\psi_y \ldots$". The upper envelope is φ^δ, not φ.
- **Correction / repair:** Replace φ by φ^δ in that sentence.

C6-M21. Remark 6.27 / Exercise 6.12: variable mismatch

- **Precise location:** printed p. 176; repeated printed p. 186, Remark 6.27 / Exercise 6.12; the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** The remark/exercise begins "Let u be a bounded plurisubharmonic function" but the assumption and regularization use φ.
- **Correction / repair:** Use a single variable, preferably φ, throughout.

C6-M23. Section 6.2.3: expression $dd^cQ \geq 0$ is not well formed

- **Precise location:** printed p. 177, Section 6.2.3; the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** The text says that $F_+(x, s, p, Q) \geq 0$ is consistent only when $dd^cQ \geq 0$. Here Q is a real quadratic form / Hessian matrix, not a function to which dd^c is applied.
- **Correction / repair:** Replace by $Q^{1,1} \geq 0$, or, in the test-function language, $dd^cq(x_0) \geq 0$.

C6-M24. Plurisubharmonic envelope definition has wrong symbols

- **Precise location:** printed p. 178, Section 6.2.3, displayed definition of the plurisubharmonic envelope $P_B(h)$.
- **Issue:** The formula reads roughly $P(h)(z) = P_Bf(z) := (\sup\{\psi(x) : \psi \in \mathrm{PSH}(B),\ \psi \leq h\})^*$. It should depend on h, not f, and the envelope at z should use $\psi(z)$, not $\psi(x)$.
- **Correction / repair:** Write

$$P_B(h)(z) := (\sup\{\psi(z) : \psi \in \mathrm{PSH}(B),\ \psi \leq h\})^* .$$

C6-M25. Lemma 6.30: the statement and proof use an unfixed point and inconsistent measure symbols

- **Precise location:** printed pp. 178–179, Lemma 6.30(1), inequality containing $f(z_0)$, and the proof of parts (1)–(2), where ν is introduced without definition.
- **Issue:** no point z_0 is fixed in item (1), and the lemma alternates between $f\beta^n$ and ν for the same right-hand side.
- **Correction / repair:** define once $\mu := f\beta^n$ and state $(dd^c\psi)^n_{\mathrm{BT}} \leq \mu$ on Ω; use μ throughout the proof. If a pointwise density inequality is intended instead, quantify it for every $z_0 \in \Omega$.

C6-M28. Theorem 6.31 proof: invalid normalization by multiplying the unknowns by a small constant

- **Precise location:** printed p. 180, Theorem 6.31 proof; the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** The proof says that since u and $\bar{u}$ are bounded, "multiplying by a small constant" one can assume the sup/inf-convolution radius constant is 1. Multiplying the unknowns does not preserve the equation $(dd^c u)^n = e^u f \beta^n$ or its sub/supersolution inequalities.
- **Correction / repair:** Do not rescale u and $\bar{u}$. Keep the constant A in the sup/inf convolutions, or reparametrize the convolution parameter so that the displayed local radius is obtained without changing the PDE.

C6-M30. Theorem 6.31 proof: wrong dimension in Jensen step

- **Precise location:** printed p. 181, Theorem 6.31 proof; the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** The argument is on $\mathbb{R}^{2n}$, but it writes $p_k \in \mathbb{R}^n$ and uses I_n in the real Hessian inequality.
- **Correction / repair:** Replace by $p_k \in \mathbb{R}^{2n}$ and I_{2n} for the real Hessian. If translating into Levi forms, then write the corresponding $dd^c|x|^2$ term separately.

C6-M31. Section 6.4: Hamiltonian for Perron's method uses φ instead of the scalar variable r

- **Precise location:** printed p. 182, Section 6.4; the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** The Hamiltonian is defined as $F(x, r, p, Q) := -(dd^c Q^{1,1})^n + e^{\varepsilon\varphi} f \beta^n$. A Hamiltonian in variables (x, r, p, Q) cannot depend on the unknown function symbol φ; it should use r. Also $dd^c Q^{1,1}$ is not the right notation for the Hermitian part of a Hessian matrix.
- **Correction / repair:** Write the finite branch as, schematically, $F(x, r, p, Q) = -\det(Q^{1,1}) + e^{\varepsilon r} f(x)$ with the book's chosen identification with β^n, and set $F = +\infty$ when $Q^{1,1}$ is not semipositive.

C6-M32. Theorem 6.32 statement: wrong boundary-function symbol in the conclusion

- **Precise location:** printed p. 182, Theorem 6.32 statement; the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** The conclusion says the viscosity solution satisfies $u = \gamma$ on $\partial\Omega$. The constructed unknown is φ.
- **Correction / repair:** Replace by $\varphi = \gamma$ on $\partial\Omega$.

C6-M33. Theorem 6.32 proof: wrong point/differential notation in the contradiction assumption

- **Precise location:** printed p. 182, Theorem 6.32 proof; the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** The contradiction assumption contains $dd^c q(x_0)$ although the point is z_0.
- **Correction / repair:** Replace x_0 by z_0.

C6-M34. Theorem 6.32: the bump calculation mixes the real Hessian and the Levi form

- **Precise location:** printed p. 183, proof of Theorem 6.32, display computing the second derivative of the quadratic bump q_δ at z_0.
- **Issue:** Q is used for the complex Levi form elsewhere, while the displayed $D^2 q_\delta = Q - 2\delta I_n$ is a real-Hessian identity in real dimension $2n$.

- **Correction / repair:** write $D^2 q_\delta(z_0) = D^2 q(z_0) - 2\delta I_{2n}$. If the intended display is instead for the Levi form, write $dd^c q_\delta = dd^c q - \delta\, dd^c|z|^2$.

C6-M36. Theorem 6.32 / Proposition 6.33: circular dependency in the pluripotential identification

- **Precise location:** printed pp. 183, Theorem 6.32 / Proposition 6.33; the exact printed wording or display is reproduced or quoted under **Issue**.

- **Issue:** Theorem 6.32 ends by saying that the Perron solution is the pluripotential solution by Proposition 6.33. But Proposition 6.33 then proves the converse direction by invoking Theorem 6.32 to say that a viscosity solution is the unique pluripotential solution on balls. This is circular as written.

- **Correction / repair:** Separate the viscosity Perron theorem from the pluripotential identification. Then prove Proposition 6.33 using local pluripotential Dirichlet existence/uniqueness from Chapter 5 plus viscosity uniqueness; after that, return to Theorem 6.32's final "coincides with" statement.

C6-M37. Exercise 6.1: dual supersolution statement needs upper semicontinuity of F

- **Precise location:** printed p. 184, Exercise 6.1; the exact printed wording or display is reproduced or quoted under **Issue**.

- **Issue:** The exercise assumes F is lower semi-continuous and asks for dual statements for supersolutions. The envelope stability statement for supersolutions requires the corresponding upper semicontinuity of the Hamiltonian.

- **Correction / repair:** Add a separate hypothesis for the dual part: F should be upper semi-continuous, and the family should be locally bounded from below, with lower semicontinuous regularization of the infimum.

C6-M38. Exercise 6.2: gluing boundary condition is on the wrong boundary

- **Precise location:** printed p. 184, Exercise 6.2; the exact printed wording or display is reproduced or quoted under **Issue**.

- **Issue:** The assumptions include $\bar\Omega_2 \subset \Omega_1$, but then ask for $u \geq v$ on $\Omega_2 \cap \partial\Omega_1$, which is empty under the inclusion. The gluing condition should be imposed on $\partial\Omega_2$.

- **Correction / repair:** Replace by the usual condition $\limsup_{\Omega_2 \ni x \to z} v(x) \leq u(z)$ for $z \in \partial\Omega_2$, or, in the continuous case, $u \geq v$ on $\partial\Omega_2$.

C6-M39. Exercise 6.4: the displayed piecewise function is not well-defined and does not satisfy the boundary condition

- **Precise location:** printed p. 184, Exercise 6.4; the exact printed wording or display is reproduced or quoted under **Issue**.

- **Issue:** The two branches overlap at $-1/2$ and $1/2$ with different values. Moreover, at $x = \pm 1$ the displayed formula gives -1, not the prescribed boundary value 0.

- **Correction / repair:** Replace by a continuous piecewise-affine example with slopes ± 1 and zero boundary values, e.g. add the missing affine constants in the outer pieces. One possible intended function is $x + 1$ on $[-1, -1/2]$, $-x$ on $[-1/2, 0]$, x on $[0, 1/2]$, and $1 - x$ on $[1/2, 1]$.

C6-M40. Exercise 6.7: shifted test function and final dense-set notation

- **Precise location:** printed p. 185, Exercise 6.7; the exact printed wording or display is reproduced or quoted under **Issue**.

- **Issue:** The exercise introduces $q(x) = A|x - x_0|^2$, but then uses $\widehat{q}$ without defining the shifted function that actually touches u at $\widehat{x}$. The final line says $D^{2,+}u(x_0)$ is dense in Ω, which is not meaningful.

- **Correction / repair:** Define $\widehat{q}(x) = q(x) + M$ (or the equivalent shifted quadratic) before using its jet. Replace the final phrase by: $D^{2,+}_\Omega u = \{x \in \Omega : J^{2,+}u(x) \neq \varnothing\}$ is dense in Ω.

C6-M41. Exercise 6.10: real and Hermitian notation are mixed

- **Precise location:** printed p. 185, Exercise 6.10; the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** The exercise assumes $\Omega \subset \mathbb{R}^N$, but then takes $H \in H_N^+$ to be Hermitian. This mixes the real and complex settings.
- **Correction / repair:** Either formulate the exercise in $\mathbb{C}^n$ with $H \in H_n^+$, or formulate it in $\mathbb{R}^N$ with H a positive real symmetric matrix.

C6-M42. Exercise 6.13: missing quantifier over p

- **Precise location:** printed p. 186, Exercise 6.13; the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** Part (2) says that for small δ, the function $x \to h(x) + < p, x >$ attains its maximum at an interior point, but it does not quantify p.
- **Correction / repair:** Insert "for every $p \in \mathbb{R}^N$ with $|p| < \delta$."

C6-M43. Exercise 6.13: Hessian lower bound has a wrong factor

- **Precise location:** printed p. 186, Exercise 6.13; the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** Part (1) gives $D^2 h \geq -kI_N$ a.e., but part (3) asks to prove $-(k/2)I_N \leq D^2 h \leq 0$. The factor 1/2 is inconsistent and would change the Lipschitz/Jensen estimate.
- **Correction / repair:** Replace $-(k/2)I_N$ by $-kI_N$ in part (3).

C6-M44. Exercise 6.14(2): the radial Levi-form bound is mis-parenthesized, and the next constant is too large

- **Precise location:** printed p. 186, Exercise 6.14(2), final display $dd^c|z| \geq |z|^{-1/4}\beta \geq \beta$.
- **Issue:** with $\beta = dd^c|z|^2$, the eigenvalues of $dd^c|z|$ relative to β are $1/(2|z|)$ tangentially and $1/(4|z|)$ radially. Thus the likely intended first coefficient is $1/(4|z|)$, not $|z|^{-1/4}$; even then, $|z| < 1$ gives only a coefficient at least $1/4$.
- **Correction / repair:**
$$dd^c|z| \geq \frac{1}{4|z|}\,\beta \geq \frac{1}{4}\,\beta \qquad (0 < |z| < 1).$$
  The stronger last bound $\geq \beta$ is valid after restricting to $|z| \leq 1/4$.

C6-N01. Section 6.1: uniform ellipticity alone does not imply a unique classical solution

- **Type:** overbroad existence/uniqueness claim.
- **Precise location:** printed p. 163, introductory paragraph on uniformly elliptic equations, sentence asserting existence of a unique C^2 solution.
- **Printed text / issue:** uniform ellipticity by itself supplies neither the coefficient regularity nor the properness, coercivity, domain, and boundary hypotheses needed for classical solvability and uniqueness.
- **Correction / repair:** qualify the sentence: classical existence and uniqueness hold only under the standard additional structural, regularity, domain, and boundary assumptions appropriate to the cited elliptic theorem.

C6-N02. Example on $1 - |u'| = 0$: the asserted contradiction is not a contradiction

- **Type:** invalid test-function argument.
- **Precise location:** printed p. 168, one-dimensional example immediately after the viscosity definitions, argument concluding from $|q'(0)| = 1$.
- **Printed text / issue:** equality $|q'(0)| = 1$ is compatible with the equation and gives no contradiction.
- **Correction / repair:** use the constant upper test $q \equiv 1$ at the contact point. Then $q'(0) = 0$ and $1 - |q'(0)| = 1 > 0$, which directly shows that the displayed u_0 is not a viscosity subsolution under the book's sign convention.

C6-N03. Semicontinuous regularizations F_ε and F^ε need not be continuous or converge to F

- **Type:** missing regularity hypothesis.
- **Precise location:** printed p. 170, paragraph defining F_ε and F^ε, assertions of continuity and convergence as $\varepsilon \downarrow 0$.
- **Printed text / issue:** for a general discontinuous F, infimum/supremum regularizations need not be continuous and converge only to the appropriate semicontinuous envelopes.
- **Correction / repair:** assume F is continuous (locally uniformly continuous on the compact sets used) to obtain local uniform convergence, or replace the conclusions by convergence to the lower and upper semicontinuous envelopes F_* and F^*.

Typographical and editorial corrections

- printed p. 176, proof using a capacity-null exceptional set: "quasi everywere" $\longrightarrow$ "quasi-everywhere".
- printed p. 168, one-dimensional boundary-value example: "with the the boundary condition" $\longrightarrow$ "with the boundary condition".
- printed p. 176, proof of Lemma 6.30(2): "a super test" should read "a lower test" (or "a test from below") for a supersolution.

Chapter 7. Compact Kähler manifolds

Range checked: printed pp. 191–214.

Mathematical corrections and proof clarifications

C7-M01. Definition 7.1: transition map domain and codomain are reversed

- **Precise location:** printed p. 191, Definition 7.1; the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** The transition map is defined as $f_{\alpha\beta} := z_\beta \circ z_\alpha^{-1}$, but the displayed domain is $z_\beta(U_\alpha \cap U_\beta)$ and the codomain is $z_\alpha(U_\alpha \cap U_\beta)$. For $z_\beta \circ z_\alpha^{-1}$, the domain should be the z_α-chart and the codomain the z_β-chart.
- **Correction / repair:** Replace the display by $f_{\alpha\beta} : z_\alpha(U_\alpha \cap U_\beta) \to z_\beta(U_\alpha \cap U_\beta)$.

C7-M02. Definition 7.4: the smoothness criterion for projective varieties is insufficient

- **Precise location:** printed p. 192, Definition 7.4; the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** The text says that V is a smooth submanifold if the rank of the matrix $(@P_i/@z_\alpha)$ is the same at each point of V. Constant rank alone is not the correct smoothness criterion for an arbitrary system of homogeneous equations; one needs the rank to equal the local codimension after passing to suitable affine coordinates/generators of the local ideal.

- **Correction / repair:** Replace by a Jacobian-rank criterion: in each affine chart, V is smooth at p if the differentials of local defining equations generate the conormal space with rank equal to $codim_p V$. If the intended statement is only for complete intersections, explicitly assume the expected rank.

C7-M03. Section 7.1.2.1: elliptic curve lattice is written with a tensor product

- **Precise location:** printed p. 193, Section 7.1.2.1; the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** The elliptic curve is written as $E_\tau = C/ZotimesZ[\tau]$. This is not the lattice of an elliptic curve. The standard lattice is $Z + \tau Z$ or $Z \oplus \tau Z$.
- **Correction / repair:** Replace by $E_\tau = C/(Z + \tau Z)$, with $\operatorname{Im} \tau > 0$.

C7-M04. Section 7.1.2.2: classification of compact Riemann surfaces is attributed to the wrong theorem

- **Precise location:** printed p. 193, Section 7.1.2.2; the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** The trichotomy $\mathbb{P}^1$, elliptic curve, or Δ/Γ is a consequence of the uniformization theorem for Riemann surfaces, not merely the Riemann mapping theorem.
- **Correction / repair:** Replace "Riemann mapping theorem" by "uniformization theorem". In the compact hyperbolic case, also say that Γ is a torsion-free co-compact discrete subgroup of $PSL(2, R)$.

C7-M05. Blow-up with smooth center: exceptional divisor is not just P^{k-1}

- **Precise location:** printed p. 194, Section 7.1.3, paragraph on the blow-up along a smooth center Y, sentence defining the exceptional divisor E.
- **Issue:** The text writes $E := \pi^{-1}(Y) \simeq \mathbb{P}^{k-1}$. For a blow-up along a positive-dimensional center, the exceptional divisor is a projective bundle over Y, namely $P(N_{Y/X})$; locally it is $(Y \cap U) \times \mathbb{P}^{k-1}$, not a single copy of $\mathbb{P}^{k-1}$.
- **Correction / repair:** Replace by $E = \pi^{-1}(Y) \simeq \mathbb{P}(N_{Y/X})$, and locally $E \cap \widetilde{U}_Y \simeq (Y \cap U) \times \mathbb{P}^{k-1}$.

C7-M06. Definition of transition functions for vector bundles: composition convention is inconsistent

- **Precise location:** printed p. 196, Definition of transition functions for vector bundles; the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** The paragraph first says that $\varphi_V \circ \varphi_U^{-1}$ induces the transition automorphism, but the displayed formula is for $\varphi_U \circ \varphi_V^{-1}$. The latter is consistent with the later convention $s_U = g_{UV} s_V$.
- **Correction / repair:** Replace "$\varphi_V \circ \varphi_U^{-1}$" by "$\varphi_U \circ \varphi_V^{-1}$", or switch the displayed formula and all later transition conventions consistently.

C7-M07. Section 7.2.1.2: operation formulas contain wrong fiber/dimension notation

- **Precise location:** printed p. 196, Section 7.2.1.2; the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** For $E \oplus F$, the fiber is written as $E_x \oplus F_y$; it should be $E_x \oplus F_x$. For $E \otimes F$, the transition function is written as lying in $\mathrm{GL}(k, \mathbb{C}^r \otimes \mathbb{C}^s)$, but k has not been defined and this is not the standard notation for the automorphism group of the tensor-product fiber.
- **Correction / repair:** Replace $E_x \oplus F_y$ by $E_x \oplus F_x$, and write $g_{UV} \otimes h_{UV} \in \mathrm{GL}(\mathbb{C}^r \otimes \mathbb{C}^s)$ or $\mathrm{GL}(rs, \mathbb{C})$.

C7-M08. Divisors and associated line bundles: local defining functions ignore multiplicities and poles

- **Precise location:** printed p. 198, Section 7.2.2.2, paragraph constructing the line bundle L_D from local equations f_U.
- **Issue:** For a general divisor $D = \sum_i a_i V_i$ with possibly negative coefficients and multiplicities, one cannot simply choose a holomorphic function f_U such that $D \cap U = \{f_U = 0\}$. That describes only the support of an effective reduced divisor. The transition $g_{UV} = f_U/f_V$ should be built from local meromorphic equations whose divisors equal $D|_U$.
- **Correction / repair:** Write: locally choose a meromorphic function f_U with $(f_U) = D|_U$; then f_U/f_V is a nowhere-vanishing holomorphic function on overlaps and defines L_D. If the authors intend only effective divisors here, state that restriction and include multiplicities.

C7-M10. Example 7.18: holomorphic 1-forms are confused with tangent-vector sections

- **Precise location:** printed p. 200, Example 7.18; the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** The text says that the holomorphic one-forms $dz_1, ..., dz_n$ descend and induce a basis of nowhere vanishing holomorphic sections of T'_X. But dz_i are sections of the holomorphic cotangent bundle $\Omega^{1,0}_X$, not of the holomorphic tangent bundle. The conclusion that T'_X is trivial is true, but its basis is given by $\partial/\partial z_i$.
- **Correction / repair:** Replace by: "The Euclidean holomorphic vector fields $\partial/\partial z_i$ descend and trivialize T'_X; equivalently the one-forms dz_i trivialize $\Omega^{1,0}_X$. Hence K_X is trivial."

C7-M12. Section 7.3.1.1: local expression for currents has bidegree/bidimension indices reversed

- **Precise location:** printed p. 203, Section 7.3.1.1; the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** The text says that a current of bidimension (p, q) acts on forms of bidegree (p, q) and hence has bidegree $(n - p, n - q)$, but then locally writes $T = \sum_{|I|=p,|J|=q} T_{I,J} dz_I \wedge dbarz_J$. The displayed differential-form indices correspond to bidegree (p, q), not $(n - p, n - q)$.
- **Correction / repair:** Either change the preceding sentence to say that T has bidegree (p, q), or keep bidimension (p, q) and write local basis forms of bidegree $(n - p, n - q)$.

C7-M13. Section 7.3.1.4: pull-back of positive closed $(1, 1)$-currents is not always defined for arbitrary holomorphic maps

- **Precise location:** printed pp. 205–206, Section 7.3.1.4; the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** The text says that one can pull back a positive closed $(1, 1)$-current $S = dd^c\varphi$ by setting $f^*S = dd^c(\varphi \circ f)$. This requires $\varphi \circ f$ not to be identically $-\infty$ on any component. If a component of X is mapped into the polar set of a local potential of S, the formula is not meaningful as a current. The final sentence "one can always pull-back the current of integration along an analytic hypersurface" has the same problem if the map is contained in the hypersurface.
- **Correction / repair:** Add the standard componentwise condition: local potentials of S must not pull back identically to $-\infty$; equivalently, no irreducible component of the source is mapped into the relevant polar set/hypersurface. Otherwise state a separate divisor-theoretic convention when it exists.

C7-M14. Section 7.4.3.1: curvature tensor is defined using the Kähler form instead of the Riemannian metric

- **Precise location:** printed p. 209, Section 7.4.3.1; the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** The Riemannian curvature tensor is defined by $R(u, v, w, x) := \omega(R(u, v)x, w)$. But ω is a skew-symmetric Kähler form, not the Riemannian scalar product. Sectional curvature later uses $R(x, y, x, y)$, which should be computed from g, not directly from ω.
- **Correction / repair:** Replace by a metric convention such as $R(u, v, w, x) := g(R(u, v)w, x)$ or $g(R(u, v)x, w)$, and adjust the following index notation consistently.

C7-M16. Section 7.4.3.2: Mok classification statement uses "sectional" where "bisectional" is intended

- **Precise location:** printed p. 210, Section 7.4.3.2; the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** The text says that if the holomorphic sectional curvature of a compact Kähler manifold is non-negative, then it is, up to an étale cover, a product of a compact Hermitian symmetric space and a torus. The cited Mok theorem is the generalized Frankel theorem for non-negative holomorphic bisectional curvature, not merely non-negative holomorphic sectional curvature.
- **Correction / repair:** Replace "holomorphic sectional curvature" by "holomorphic bisectional curvature" in this bullet.

C7-M18. Exercise 7.8: Picard group of a blow-up uses direct sum, not tensor product with $\mathbb{Z}$

- **Precise location:** printed p. 212, Exercise 7.8; the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** The exercise states $\mathrm{Pic}(\widetilde{X}_Y) \simeq \pi^* \mathrm{Pic}(X) \otimes \mathbb{Z}$. The exceptional divisor contributes a free $\mathbb{Z}$ summand; tensor product is the wrong group operation and gives the wrong object.
- **Correction / repair:** Replace by $\mathrm{Pic}(\widetilde{X}_Y) \simeq \pi^* \mathrm{Pic}(X) \oplus \mathbb{Z}[E]$, under the usual hypotheses, with $[E]$ the exceptional divisor class.

C7-N01. Section 7.2.2: the lower section-growth bound cannot hold for every tensor power

- **Type:** false quantifier over k.
- **Precise location:** printed p. 199, paragraph giving $C^{-1}k^{\kappa} \leq h^0(X, L^k)$.
- **Printed text / issue:** take $X = \mathbb{P}^1 \times E$ and $L = \mathrm{pr}_1^*\mathcal{O}(1) \otimes \mathrm{pr}_2^* T$, where T is a nontrivial torsion line bundle of order m on the elliptic curve E. Then $h^0(X, L^k) = 0$ unless $m \mid k$.
- **Correction / repair:** retain the polynomial upper bound for all k, but state the positive lower bound only for sufficiently divisible powers (or along a suitable infinite subsequence).

C7-N02. Blow-up exercise: "holomorphic self-maps" must be restricted

- **Type:** false classification statement.
- **Precise location:** printed p. 213, exercise concerning self-maps of the blow-up of $\mathbb{P}^2$ at a point p.
- **Printed text / issue:** constant maps are holomorphic self-maps but do not correspond to projective automorphisms fixing p.
- **Correction / repair:** replace "holomorphic self-maps" by "biholomorphic automorphisms." These correspond to automorphisms of $\mathbb{P}^2$ fixing the center p.

Typographical and editorial corrections

- printed p. 212, "induces and automorphism" $\longrightarrow$ "induces an automorphism".
- printed p. 208, definition of the tangent bundle: "holomophic tangent bundle" $\longrightarrow$ "holomorphic tangent bundle".
- printed p. 211, notation paragraph: "with no dange of confusion" $\longrightarrow$ "with no danger of confusion".

Chapter 8. Quasi-plurisubharmonic functions

Range checked: printed pp. 215–234.

Mathematical corrections and proof clarifications

C8-M02. Proposition 8.5 proof: the non-smooth measure argument mislabels a function as a sequence

- **Precise location:** printed p. 218, proof of Proposition 8.5; the exact printed wording or display is reproduced or quoted under **Printed text**.
- **Printed text:** After setting
$$\psi = \sum_{j\geq 1} 2^{-j}\psi_j,$$
the book says: "This is a decreasing sequence of ω-psh functions."
- **Issue:** ψ is a function. What is decreasing is the sequence of partial sums
$$\psi^{(N)} := \sum_{j=1}^{N} 2^{-j}\psi_j,$$
since each $\psi_j \leq 0$. The intended argument is that $\psi^{(N)} \downarrow \psi$, hence $\psi \in \mathrm{PSH}(X,\omega)$ or $\psi \equiv -\infty$.
- **Correction / repair:** Replace the sentence by: "The partial sums form a decreasing sequence of ω-psh functions."

C8-M03. Lelong class compactification formula should use limsup, not an ordinary limit

- **Precise location:** printed p. 219, Section 8.2.1.1; the exact printed wording or display is reproduced or quoted under **Printed text**.
- **Printed text:** The value on H_∞ is defined using
$$\lim_{y\in\mathbb{C}^n\to x} \left(\psi(y) - \frac{1}{2}\log(1+|y|^2)\right).$$
- **Issue:** For a general $\psi \in \mathcal{L}(\mathbb{C}^n)$, this ordinary limit need not exist. The extension to $\mathbb{P}^n$ is obtained by upper semicontinuous regularization, hence by a $\limsup$.
- **Correction / repair:** Replace lim by
$$\limsup_{y\in\mathbb{C}^n,\, y\to x} \left(\psi(y) - \frac{1}{2}\log(1+|y|^2)\right).$$

C8-M04. Metric associated to a section: the zero section should be excluded

- **Precise location:** printed p. 219, after Definition 8.7; the exact printed wording or display is reproduced or quoted under **Printed text**.
- **Printed text:** "if $s = \{s_\alpha\}$ is a holomorphic section of L, then $\psi = \{\psi_\alpha := \log |s_\alpha|\}$ is a positive singular metric."
- **Issue:** If $s \equiv 0$, then $\log |s_\alpha| \equiv -\infty$, which is not an L^1-weight in the sense of Definition 8.7.
- **Correction / repair:** Write "if s is a non-zero holomorphic section".

C8-M05. Definition of PSH^+ is recursive

- **Precise location:** printed p. 220, before the proof of Proposition 8.8; the exact printed wording or display is reproduced or quoted under **Printed text**.
- **Printed text:**
$$\mathrm{PSH}^+(V, \omega) := \bigcup_{0<\varepsilon<1} \mathrm{PSH}^+(V, (1-\varepsilon)\omega).$$
- **Issue:** The right-hand side repeats PSH^+, making the definition recursive.
- **Correction / repair:** Replace the right-hand side by
$$\bigcup_{0<\varepsilon<1} \mathrm{PSH}(V, (1-\varepsilon)\omega).$$

C8-M07. Theorem 8.9: the Ohsawa–Takegoshi lower estimate should be stated for large j

- **Precise location:** printed p. 222, proof of Theorem 8.9; the exact printed wording or display is reproduced or quoted under **Printed text**.
- **Printed text:** "there exist $j_0 \in \mathbb{N}$ and $C_4 > 0$ large enough so that $\forall x \in X, \forall j \in \mathbb{N}$, there exists $s \in \Gamma(X, L^j)$ . . . "
- **Issue:** The auxiliary weight is $h_{j,j_0} = (j - j_0)\psi + j_0 h$. The proof only makes sense for $j \geq j_0$, or at least for all sufficiently large j.
- **Correction / repair:** Replace $\forall j \in \mathbb{N}$ by $\forall j \geq j_0$ or $\forall j \gg 1$.

C8-M08. Theorem 8.9: the random-phase conclusion needs a logarithmic selection lemma

- **Precise location:** printed p. 222, last paragraph and final display of the proof of Theorem 8.9, from *We can now approximate φ_j by $\widetilde{\varphi}_j$* to the end-of-proof symbol.
- **Issue:** the proof establishes only the torus average identity for $|S_{\theta,j}|^2$. The desired conclusion concerns $j^{-1} \log |S_{\theta,j}|$ in $L^1(X)$; an average of squared norms does not by itself select one phase with the required control of the negative part of the logarithm.
- **Repair needed:** insert a proved phase-selection lemma controlling the negative logarithmic tail (or cite an applicable random-section theorem), and combine it with Fubini and compactness to choose phases θ_j for which $j^{-1} \log |S_{\theta_j,j}| - \varphi_{j,j_0} \to 0$ in $L^1(X)$. The printed L^2 identity alone is insufficient; no particular quantitative torus-logarithm bound is asserted here without proof.

C8-M11. The bound for bounded potentials misses the factor n

- **Precise location:** printed p. 226, paragraph after Proposition 8.14; the exact printed wording or display is reproduced or quoted under **Printed text**.
- **Printed text:**
$$\|\nabla\varphi\|_{L^2}^2 \leq \frac{\mathrm{Vol}_\omega(X)}{2}.$$
- **Issue:** The book has just stated
$$|\nabla_\omega\varphi|^2\omega^n = n\, d\varphi \wedge d^c\varphi \wedge \omega^{n-1}.$$
From
$$\omega + dd^c(\varphi^2) \geq 2d\varphi \wedge d^c\varphi$$
one gets
$$2\int_X d\varphi \wedge d^c\varphi \wedge \omega^{n-1} \leq \mathrm{Vol}_\omega(X),$$
hence
$$\|\nabla\varphi\|_{L^2}^2 \leq \frac{n}{2}\mathrm{Vol}_\omega(X).$$
- **Correction / repair:** Insert the factor n, unless the L^2-norm is being normalized in a nonstandard way not stated here.

C8-M12. Proposition 8.15: e^{ψ_D} is generally not smooth

- **Precise location:** printed p. 226, proof of Proposition 8.15; the exact printed wording or display is reproduced or quoted under **Printed text**.
- **Printed text:** "ψ_D has poles along D and e^{ψ_D} is smooth."
- **Issue:** With the convention used in the book, a divisor potential is locally $\log|f| + O(1)$, so $e^{\psi_D} \sim |f|$, which is not smooth across D in general. The smooth object is $|f|^2e^{-2h}$, i.e. $e^{2\psi_D}$ if $\psi_D = \log|s|_h$, or else one should use the convention $\psi_D = \log|s|_h^2$.
- **Correction / repair:** Replace by "$e^{2\psi_D}$ is smooth" or redefine ψ_D with the squared norm.

C8-M14. Proposition 8.16: reduction to the bounded case needs a diagonal argument

- **Precise location:** printed p. 228, proof of Proposition 8.16; the exact printed wording or display is reproduced or quoted under **Printed text**.
- **Printed text:** "We can assume without loss of generality that φ is bounded (replacing φ by $\max(\varphi, -j)$)."
- **Issue:** Replacing φ by its truncations gives a family of bounded functions, not immediately a single decreasing smooth sequence converging to the original φ. One still needs a diagonal construction preserving monotonicity.
- **Correction / repair:** After proving the bounded case, approximate each $\max(\varphi, -j)$, then choose a diagonal sequence u_j with
$$\max(\varphi, -j) - 2^{-j} \leq u_j \leq \max(\varphi, -j+1) + 2^{-j}$$
and replace by successive regularized minima/maxima as needed to obtain a decreasing sequence.

C8-M20. Exercise 8.3 is false for non-biholomorphic maps

- **Precise location:** printed p. 232, Exercise 8.3; the exact printed wording or display is reproduced or quoted under **Printed text**.
- **Printed text:** For a holomorphic map $f : X \to Y$, the exercise asks to show

$$\mathrm{PSH}(X, f^*\omega_Y) = f^*\mathrm{PSH}(Y, \omega_Y).$$

- **Issue:** This is false in general. For example take $f : \mathbb{P}^1 \to \mathbb{P}^1$, $f([z_0 : z_1]) = [z_0^2 : z_1^2]$. Then $f^*\omega_{FS}$ represents $2\omega_{FS}$, so $\mathrm{PSH}(\mathbb{P}^1, f^*\omega_{FS})$ contains many potentials with asymmetric singularities, whereas pullbacks from the target are invariant under the deck symmetry $z \mapsto -z$.
- **Correction / repair:** Replace the exercise by the true inclusion

$$f^*\mathrm{PSH}(Y, \omega_Y) \subset \mathrm{PSH}(X, f^*\omega_Y),$$

with the usual caveat that the pullback is not identically $-\infty$. Equality requires strong additional hypotheses and is not true for finite maps of degree > 1.

C8-M21. Exercise 8.11 gives the wrong scaling law for Tian's alpha invariant

- **Precise location:** printed p. 233, Exercise 8.11; the exact printed wording or display is reproduced or quoted under **Printed text**.
- **Printed text:**

$$\alpha(\{\lambda\omega\}) = \lambda\alpha(\{\omega\}), \qquad \lambda > 0.$$

- **Issue:** The scaling is inverse. Since

$$\mathrm{PSH}(X, \lambda\omega) = \lambda\,\mathrm{PSH}(X, \omega),$$

normalized potentials scale by λ, and

$$e^{-a(\lambda\varphi)} = e^{-(a\lambda)\varphi}.$$

Thus the integrability threshold satisfies

$$\alpha(\{\lambda\omega\}) = \lambda^{-1}\alpha(\{\omega\}).$$

- **Correction / repair:** Replace $\lambda\alpha(\{\omega\})$ by $\lambda^{-1}\alpha(\{\omega\})$.

C8-N01. Proposition 8.4: the compactness dichotomy is false for the full sequence

- **Type:** missing subsequence clause.
- **Precise location:** printed p. 217, Proposition 8.4, first assertion about a sequence of quasi-plurisubharmonic functions; PDF p. 242.
- **Printed text / issue:** alternating $\varphi_{2m} = 0$ and $\varphi_{2m+1} = -m$ satisfies neither alternative for the full sequence.
- **Correction / repair:** insert “after passing to a subsequence” before the dichotomy. Alternatively, assume a uniform lower bound for $\sup_X \varphi_j$, which rules out uniform convergence to $-\infty$.

C8-N02. Pullback through the Kodaira map has the wrong background form

- **Type:** normalization error.
- **Precise location:** printed p. 220, argument pulling an ω_{FS}-psh function back through Φ_k and declaring the result ω-psh.
- **Printed text / issue:** $\psi \circ \Phi_k$ is $\Phi_k^*\omega_{\mathrm{FS}}$-psh, not automatically ω-psh.
- **Correction / repair:** if $\theta_k := \Phi_k^*\omega_{\mathrm{FS}} = k\omega + dd^c\rho_k$, use

  $$k^{-1}\big(\psi \circ \Phi_k + \rho_k\big),$$

  which is ω-psh.

Typographical and editorial corrections

C8-E01. Lemma 8.17: clarify the curvature-bound wording

- **Precise location:** printed p. 230, Lemma 8.17, description of the constant A.
- **Correction / repair:** replace "A is a lower bound of the negative part" by either "$-A$ is a lower bound for the bisectional curvature" or "A is an upper bound for its negative part."

C8-T01. Typo: "straighforward"

- **Precise location:** printed p. 217, proof of Proposition 8.4; the exact printed wording or display is reproduced or quoted under **Correction / repair**.
- **Correction / repair:** "straightforward".

C8-T02. Heading typo: "holomorphic lines bundles"

- **Precise location:** printed p. 218, Section 8.2.1 heading; the exact printed wording or display is reproduced or quoted under **Correction / repair**.
- **Correction / repair:** "holomorphic line bundles".

C8-T03. Product notation omitted in local trivialization

- **Precise location:** printed p. 219, Section 8.2.1.2; the exact printed wording or display is reproduced or quoted under **Printed text**.
- **Printed text:** $\Phi_\alpha : \pi^{-1}(U_\alpha) \to U_\alpha\mathbb{C}$, and later $\{x\}\mathbb{C}$.
- **Correction / repair:** Insert $\times$: $U_\alpha \times \mathbb{C}$, $\{x\} \times \mathbb{C}$.

C8-T04. Capitalization: "french"

- **Precise location:** printed p. 219, footnote 1; the exact printed wording or display is reproduced or quoted under **Correction / repair**.
- **Correction / repair:** "French".

C8-T05. Proposition 8.16 grammar

- **Precise location:** printed p. 228; the exact printed wording or display is reproduced or quoted under **Printed text**.
- **Printed text:** "there exists smooth ω-psh functions".
- **Correction / repair:** "there exist smooth ω-psh functions".

C8-T07. Lemma 8.17 grammar

- **Precise location:** printed p. 230; the exact printed wording or display is reproduced or quoted under **Printed text**.
- **Printed text:** "The crucial estimate . . . , coupled with Kiselman's theorem, provide a lemma. . . "
- **Correction / repair:** "provides".

C8-T08. Punctuation error

- **Precise location:** printed p. 225; the exact printed wording or display is reproduced or quoted under **Printed text**.
- **Printed text:** "a semi-ample line bundle. and h is a smooth metric. . . "
- **Correction / repair:** "a semi-ample line bundle, and h is. . . "

C8-T09. Subject-verb agreement

- **Precise location:** printed p. 225; the exact printed wording or display is reproduced or quoted under **Printed text**.
- **Printed text:** "its values plays an important role".
- **Correction / repair:** "its values play an important role" or "its value plays an important role".

C8-T10. Exercise 8.2 wording

- **Precise location:** printed p. 232, Exercise 8.2(4); the exact printed wording or display is reproduced or quoted under **Printed text**.
- **Printed text:** "Characterize, in term of the curvature current. . . "
- **Correction / repair:** "in terms of".

Chapter 9. Envelopes and capacities

Range checked: printed pp. 235–256.

Mathematical corrections and proof clarifications

C9-M01. Section 9.1.1: trace measure and mass norm use ω although ω is only semi-positive

- **Precise location:** printed p. 235, Section 9.1.1; the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** The chapter assumes only that ω is semi-positive and big. The text defines

$$\|T\| := \int_X T \wedge \omega^{n-p}$$

  and says that this controls the total variation of the coefficients of a positive closed (p, p)-current T. It also says that integrability with respect to all coefficients of T is equivalent to integrability with respect to the trace measure $T \wedge \omega^{n-p}$. This is true for a Kähler form, but it is not true for a merely semi-positive big form: ω can degenerate, and $T \wedge \omega^{n-p}$ need not dominate the coefficient measures of T.
- **Correction / repair:** Either assume ω is Kähler in this paragraph, or introduce a fixed background Kähler form Ω and use $T \wedge \Omega^{n-p}$ for trace/integrability estimates. Keep $\int_X T \wedge \omega^{n-p}$ only as a cohomological mass where appropriate.

C9-M02. The sentence "For $T = 0$ and $j = n$" should use the unit current

- **Precise location:** printed p. 236, Section 9.1.1, sentence immediately after the inductive definition of $dd^c u_1 \wedge \cdots \wedge dd^c u_j \wedge T$, beginning *For $T = 0$ and $j = n$.*
- **Issue:** The text says: "For $T = 0$ and $j = n$ one obtains the (unormalized) complex Monge–Ampère operator." If T is the zero current, the inductive construction gives zero.
- **Correction / repair:** Replace by "For $p = 0$, $T = 1$, and $j = n$" or "starting from the unit $(0,0)$-current".

C9-M03. Proposition 9.4 and Corollary 9.5: the $2\sup\psi$ term is false when $\sup\psi < 0$

- **Precise location:** printed pp. 237–238, Proposition 9.4 and Corollary 9.5; the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** Proposition 9.4 contains

$$\|\psi\|_{L^1(T\wedge\omega_\varphi)} \le \|\psi\|_{L^1(T)} + \big(2\sup_X \psi + \sup_X \varphi - \inf_X \varphi\big)\|T\|.$$

  This cannot be correct for arbitrary ψ. If $\psi \equiv -1000$ and φ is constant, the two L^1-norms are equal, but the extra term is negative. Corollary 9.5 inherits the same problem through the factor $1 + 2\sup_X \psi$.

- **Correction / repair:** Replace $2\sup_X \psi$ by a nonnegative term, for example

$$2\max\{\sup_X \psi, 0\}$$

  or, less sharply, $2|\sup_X \psi|$. With this correction, the proof by decomposing $\psi = \psi_0 + \sup_X \psi$, $\psi_0 \le 0$, becomes valid. Corollary 9.5 should be updated consistently.

C9-M04. Proposition 9.7(1): missing normalization in the volume lower bound

- **Precise location:** printed p. 238, Proposition 9.7(1); the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** The statement says $\mathrm{Vol}_\omega \le \mathrm{Cap}_\omega$. But in this chapter

$$V_\omega = \int_X \omega^n, \qquad MA(0) = V_\omega^{-1}\omega^n.$$

  Thus the lower bound obtained by testing with the constant function is the normalized volume probability measure, not the unnormalized measure ω^n.

- **Correction / repair:** Write

$$V_\omega^{-1}\omega^n \le \mathrm{Cap}_\omega$$

  as set functions, or explicitly define Vol_ω there as the normalized volume measure.

C9-M05. Proposition 9.7(3): the scaling inequality for normalized capacity is reversed

- **Precise location:** printed p. 238, Proposition 9.7(3); the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** The book states, for $A \ge 1$,

$$\mathrm{Cap}_\omega(\cdot) \le \mathrm{Cap}_{A\omega}(\cdot) \le A^n\,\mathrm{Cap}_\omega(\cdot).$$

  This is compatible with an unnormalized capacity, but Definition 9.6 uses normalized Monge–Ampère measures. If $\psi \in \mathrm{PSH}(X, A\omega)$ and $0 \le \psi \le 1$, then $\psi/A \in \mathrm{PSH}(X,\omega)$ and

$$MA_{A\omega}(\psi) = MA_\omega(\psi/A),$$

  hence $\mathrm{Cap}_{A\omega} \le \mathrm{Cap}_\omega$. Conversely, testing the same ω-psh functions gives

$$A^{-n}\,\mathrm{Cap}_\omega \le \mathrm{Cap}_{A\omega}.$$

- **Correction / repair:** Replace the displayed inequality by

$$A^{-n}\operatorname{Cap}_\omega(\cdot) \le \operatorname{Cap}_{A\omega}(\cdot) \le \operatorname{Cap}_\omega(\cdot).$$

The final comparability for two Kähler forms remains true after adjusting constants.

C9-M06. Proposition 9.8: proof silently switches between normalized and unnormalized capacities

- **Precise location:** printed pp. 239–240, Proposition 9.8; the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** The proof estimates integrals such as $\int_E \omega_\varphi^n$, while $\operatorname{Cap}_\omega$ was defined using $MA(\varphi) = V_\omega^{-1}\omega_\varphi^n$. Since Proposition 9.8 is only a comparison up to constants, the missing factor can be absorbed, but the displayed proof is not literally consistent with Definition 9.6.
- **Correction / repair:** Insert the factors V_ω^{-1}, or say explicitly that all normalizing constants are absorbed into the constants C, C'.

C9-M08. Corollary 9.12: the initial exceptional set is not included in the final open set

- **Precise location:** printed p. 242, Corollary 9.12; the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** The proof first chooses

$$O^{(0)} = \{\varphi < -t\}, \qquad \operatorname{Cap}_\omega(O^{(0)}) < \varepsilon/2,$$

and works on $X \setminus O^{(0)}$, replacing φ by $\varphi_t = \max(\varphi, -t)$. Later it defines

$$O_\varepsilon = \bigcup_{j\ge1} O_j$$

but does not include $O^{(0)}$. Without excluding $O^{(0)}$, uniform convergence to φ_t only proves continuity of φ_t, not of φ.

- **Correction / repair:** Use distinct notation and set

$$O_\varepsilon = O^{(0)} \cup \bigcup_{j\ge1} O_j.$$

Adjust the capacity budget so that the total is still less than ε.

C9-M10. Proposition 9.14: the vanishing set is too large for an arbitrary Borel set

- **Precise location:** printed p. 243, statement of Proposition 9.14 and its proof, especially the first sentence after the Choquet-lemma paragraph: *Let $a \in \Omega_E \setminus E$ and fix a small ball $B \Subset \Omega_E$ centered at a.*
- **Issue:** to keep the balayaged competitors equal to -1 on E, the ball must be disjoint from E. Such a ball is available from $a \notin \overline{E}$, not merely from $a \notin E$. Correspondingly, for a nonclosed dense Borel set the printed conclusion can fail: then $h^*_{E,\omega} = -1$, while $MA(h^*_{E,\omega})$ may charge $X \setminus E$.
- **Correction / repair:** for arbitrary Borel E, replace the conclusion by $MA(h^*_{E,\omega}) = 0$ on $\Omega_E \setminus \overline{E}$, and in the proof choose $B \Subset \Omega_E \setminus \overline{E}$. Alternatively, retain $\Omega_E \setminus E$ under the added hypothesis that E is closed (in particular, compact).

C9-M13. Theorem 9.15: $\mathrm{MA}(h_E)$ should be $\mathrm{MA}(h_E^*)$

- **Precise location:** printed p. 244, Theorem 9.15; the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** In the arbitrary-set case, the text says

$$(-h_{G_j})MA(h_{G_j}) \to (-h_E^*)MA(h_E)$$

  in the weak sense. For arbitrary E, the raw envelope h_E need not be usc or psh, so $MA(h_E)$ is not defined.

- **Correction / repair:** Replace $MA(h_E)$ by $MA(h_E^*)$. The following displayed formula already uses $MA(h_E^*)$.

C9-M14. Theorem 9.15: $(-h_{K_j})^*$ has the wrong star placement

- **Precise location:** printed p. 245, Theorem 9.15; the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** In the proof of continuity along decreasing compact sets, the text writes

$$(-h_{K_j})^* MA(h_{K_j}^*) \to (-h_K^*)MA(h_K^*).$$

  The factor should be $-h_{K_j}^*$, not the usc regularization of $-h_{K_j}$.

- **Correction / repair:** Replace $(-h_{K_j})^*$ by $-h_{K_j}^*$.

C9-M15. Theorem 9.17 proof: the Choquet sequence need not satisfy $\varphi_j = 0$ on K

- **Precise location:** printed p. 246, Theorem 9.17 proof; the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** The proof says Choquet's lemma gives $\varphi_j \in \mathrm{PSH}(X, \omega)$ with $\varphi_j = 0$ on K. The admissible condition in Definition 9.16 is only $\varphi_j \le 0$ on K, and pointwise equality on all of K is generally unavailable.
- **Correction / repair:** Replace "$= 0$ on K" by "≤ 0 on K". The subsequent construction still works: if $\sup_X \varphi_j \ge 2^j$, then for $x \in K$,

$$\varphi_j(x) - \sup_X \varphi_j \le -2^j.$$

C9-M17. Example 9.23: a logarithm is missing on the left-hand side

- **Precise location:** printed p. 250, Example 9.23; the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** The text says a computation yields

$$|h(\zeta)| \le \log\bigl(|z| + \sqrt{|z|^2 + 1}\bigr),$$

  which is false already at $z = 0$. The next displayed estimate for the extremal function shows that the logarithm should apply to $|h(\zeta)|$, not to the right-hand comparison alone.

- **Correction / repair:** Replace by either

$$\log|h(\zeta)| \le \log\bigl(|z| + \sqrt{|z|^2 + 1}\bigr),$$

  or by the non-logarithmic inequality

$$|h(\zeta)| \le |z| + \sqrt{|z|^2 + 1}.$$

C9-M18. Theorem 9.24 proof: impossible trivializing-cover assumption

- **Precise location:** printed p. 250, Theorem 9.24 proof; the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** The proof says one can assume $B \subset U_{\alpha_0}$ but $B \cap U_\beta = \varnothing$ for all $\beta \neq \alpha_0$. This cannot generally be arranged for a fixed open cover of X, and it is unnecessary.
- **Correction / repair:** Choose B with $\operatorname{supp} \chi \Subset U$, where U is a trivializing open set for L, fix a local frame e, and view $\chi e^{\otimes N}$, extended by zero outside U, as the smooth global section used in the $\bar\partial$-argument.

C9-M21. Proposition 9.25 proof: $D^2\varphi$ should be D^2u

- **Precise location:** printed p. 252, Proposition 9.25 proof; the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** After setting $u = P(h)$, the proof says singular integral theory gives the full real Hessian $D^2\varphi$, but the function under discussion is u.
- **Correction / repair:** Replace $D^2\varphi$ by D^2u.

Typographical and editorial corrections

- printed p. 236, Section 9.1.1, same sentence as C9-M02: “unormalized” $\longrightarrow$ “unnormalized”.

Chapter 10. Finite energy classes

Range checked: printed pp. 257–286.

Mathematical corrections and proof clarifications

C10-M01. Proposition 10.5: the cited convergence theorem is not the Lebesgue dominated convergence theorem

- **Precise location:** printed p. 259, Proposition 10.5; the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** After proving setwise convergence

$$MA(\varphi)(B) = \lim_{j\to\infty} MA(\varphi_j)(B)$$

  for Borel sets, the proof says that (10.1.2) follows “by using the Lebesgue dominated convergence theorem.” The measures vary with j, so this is not an application of the usual dominated convergence theorem.
- **Correction / repair:** Replace this sentence by a setwise-convergence argument: first prove the convergence for indicators of Borel sets, then for simple functions, and finally for bounded Borel functions by uniform approximation by simple functions.

C10-M05. Lemma 10.21 / Exercise 10.8: “multilinear” is not literally true for the potentials as written

- **Precise location:** printed pp. 267–268, Lemma 10.21, and printed p. 283, Exercise 10.8; the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** The text says the mapping

$$(\varphi_1, \ldots, \varphi_n) \mapsto \mu(\varphi_1, \ldots, \varphi_n)$$

  is multilinear. As written this is not literally a multilinear map of potentials: $\varphi_i \in PSH(X, \omega_i)$ does not form a vector space with a fixed background, and adding potentials changes the background form needed for positivity.

- **Correction / repair:** State multilinearity for the associated positive currents
$$T_i = \omega_i + dd^c\varphi_i,$$
or for pairs (ω_i, φ_i) under the operation $(\omega_1, \varphi_1) + (\omega_2, \varphi_2) = (\omega_1 + \omega_2, \varphi_1 + \varphi_2)$.

C10-M06. Proposition 10.22: the polynomial $P(t)$ is written as if it were both a measure and a scalar

- **Precise location:** printed p. 268, Proposition 10.22; the exact printed wording or display is reproduced or quoted under **Issue**.

- **Issue:** The proof defines
$$P(t_1, \dots, t_n) := \mu\Big(\sum_i t_i\varphi_i\Big),$$
then writes
$$P(t_1, \dots, t_n) = (t_1 + \cdots + t_n)^n.$$
The left side is a measure, while the right side is a scalar. The argument is intended to concern the total mass of the mixed measure.

- **Correction / repair:** Define
$$P(t) := \mu\Big(\sum_i t_i\varphi_i\Big)(X)$$
on the simplex, or better
$$P(t) := V^{-1}\int_X \Big\langle \big(\sum_i t_i(\omega + dd^c\varphi_i)\big)^n \Big\rangle.$$

C10-M08. Proposition 10.26: "have gradient in L^2" requires ω Kähler or a background metric

- **Precise location:** printed p. 270, Proposition 10.26; the exact printed wording or display is reproduced or quoted under **Issue**.

- **Issue:** The chapter assumes ω is semi-positive and big, not necessarily Kähler. The proof only gives
$$\int_X d\varphi \wedge d^c\varphi \wedge \omega^{n-1} < \infty.$$
If ω degenerates, this does not imply that the gradient is in L^2 with respect to a fixed Kähler metric.

- **Correction / repair:** Either assume ω is Kähler in Proposition 10.26, or replace the conclusion by the precise weighted Dirichlet finiteness
$$\int_X d\varphi \wedge d^c\varphi \wedge \omega^{n-1} < \infty.$$

C10-M09. Proposition 10.28 proof: derivative formula for G has the wrong evaluation mark

- **Precise location:** printed p. 271, Proposition 10.28 proof; the exact printed wording or display is reproduced or quoted under **Issue**.

- **Issue:** The proof writes
$$g'(t) = \left.\frac{dG(\varphi + tv)}{dt}\right|_{t=0} = \cdots .$$
The left side is $g'(t)$, so the derivative should be evaluated at the current parameter t, not at 0.

- **Correction / repair:** Replace by
$$g'(t) = \frac{d}{dt}G(\varphi + tv) = \int_X v\, MA(\varphi + tv),$$
or write the displayed formula only for $g'(0)$.

C10-M10. Lemma 10.29 proof: the layer-cake constant is inconsistent with the normalized MA

- **Precise location:** printed p. 272, Lemma 10.29 proof; the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** The proof uses normalized measures $MA(\varphi) = V^{-1}\omega_\varphi^n$, but then writes
$$-\int_X \varphi\, MA(\varphi) = \int_1^{+\infty} MA(\varphi)(\varphi \le -t)\, dt + \int_X \omega^n.$$
If $\varphi \le -1$, the contribution from $0 < t < 1$ is 1, not $\int_X \omega^n = V$, for normalized MA.
- **Correction / repair:** Replace $\int_X \omega^n$ by 1, or write the whole formula with unnormalized Monge–Ampère measures consistently.

C10-M13. Theorem 10.30: the quasi-triangle inequality has the wrong second term

- **Precise location:** printed p. 274, Theorem 10.30; the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** The theorem states
$$c_n I(\varphi_1, \varphi_2) \le I(\varphi_1, \varphi_3) + I(\varphi_3, \varphi_1).$$
Since I is symmetric, the right side is just twice $I(\varphi_1, \varphi_3)$. Taking $\varphi_3 = \varphi_1$ and $\varphi_2 \ne \varphi_1$ makes the right side zero, so the statement is false.
- **Correction / repair:** Replace the second term by $I(\varphi_3, \varphi_2)$:
$$c_n I(\varphi_1, \varphi_2) \le I(\varphi_1, \varphi_3) + I(\varphi_3, \varphi_2).$$
This is also the form used in the proof.

C10-M14. Lemma 10.31: the hypotheses should be in $\mathcal{E}^1$, not merely $\mathcal{E}$

- **Precise location:** printed p. 274, Lemma 10.31; the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** The lemma states the estimate for $\varphi_1, \varphi_2, \psi \in \mathcal{E}(X, \omega_0)$. But the functional $I(\cdot, \cdot)$ and the Dirichlet quantities used in the proof are defined in this section for $\mathcal{E}^1$. For general $\mathcal{E}$-potentials these quantities may be infinite or undefined.
- **Correction / repair:** Replace $\mathcal{E}(X, \omega_0)$ by $\mathcal{E}^1(X, \omega_0)$, or explicitly state that the inequality is meant in the extended sense after first proving all terms are well defined.

C10-M17. Definition 10.40 discussion: the integration-by-parts identity is too broad as stated

- **Precise location:** printed p. 279, Definition 10.40 discussion; the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** The text writes
$$\int_B -\varphi\, dd^c\varphi \wedge \theta = \int_B d\varphi \wedge d^c\varphi \wedge \theta$$
"for all test forms θ." For an arbitrary test form there are extra terms involving derivatives of θ. The displayed identity is valid when θ is closed and compactly supported, or after writing the correct integration-by-parts formula with boundary/derivative terms controlled.
- **Correction / repair:** Replace "for all test forms θ" by "for all compactly supported closed test forms θ of the appropriate bidegree", or give the full local integration-by-parts formula.

C10-M22. Exercise 10.1(ii): the normalization should be with respect to ω, not μ

- **Precise location:** printed p. 282, Exercise 10.1(ii); the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** The exercise defines
$$\varphi = \int_X g_a \, d\mu(a)$$
with $\int_X g_a \omega = 0$. Therefore the natural normalization is
$$\int_X \varphi \, \omega = 0.$$
The printed text instead says $\int_X \varphi \, d\mu = 0$, which is generally false; it is a Green energy, not the normalization forced by the definition of g_a.
- **Correction / repair:** Replace $\int_X \varphi \, d\mu = 0$ by $\int_X \varphi \, \omega = 0$.

C10-M23. Exercise 10.7(i): $MA(\varphi)$ should be $MA(\rho)$

- **Precise location:** printed p. 282, Exercise 10.7(i); the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** The exercise constructs a function ρ with logarithmic pole at x, then says: "Such a function ρ has a well-defined Monge–Ampère measure $MA(\varphi)$ and the latter has a Dirac mass at x." The measure should be that of ρ, not φ.
- **Correction / repair:** Replace $MA(\varphi)$ by $MA(\rho)$.

Typographical and editorial corrections

C10-T01. Remark 10.3: Exercice should be Exercise

- **Precise location:** printed p. 258, Remark 10.3; the exact printed wording or display is reproduced or quoted under **Correction / repair**.
- **Correction / repair:** Replace $Exercice 10.7$ by $Exercise 10.7$.

C10-T02. Corollary 10.8 proof: grammar issue

- **Precise location:** printed p. 261, Corollary 10.8 proof; the exact printed wording or display is reproduced or quoted under **Printed text**.
- **Printed text:** $Bythepreviouspropositionthatu := \max(...) belongstoE(X, \omega)$.
- **Correction / repair:** Replace by $Bythepreviousproposition, u := \max(...) belongstoE(X, \omega)$.

C10-T03. Lemma 10.12: duplicated then

- **Precise location:** printed p. 263, Lemma 10.12; the exact printed wording or display is reproduced or quoted under **Printed text**.
- **Printed text:** Moreover, if this holds then for all Borel subsets B, then ...
- **Correction / repair:** Delete the second then.

C10-T05. Theorem 10.18 footnote: necessary should be necessarily

- **Precise location:** printed p. 266, Theorem 10.18 footnote; the exact printed wording or display is reproduced or quoted under **Correction / repair**.
- **Correction / repair:** Replace The convergence necessary holds by The convergence necessarily holds.

C10-T08. Definition 10.32: subject-verb agreement

- **Precise location:** printed p. 276, Definition 10.32; the exact printed wording or display is reproduced or quoted under **Printed text**.
- **Printed text:** such that E become continuous
- **Correction / repair:** Replace by such that E becomes continuous.

C10-T09. Definition 10.42: to weak should be too weak

- **Precise location:** printed p. 280, Definition 10.42; the exact printed wording or display is reproduced or quoted under **Correction / repair**.
- **Correction / repair:** Replace slightly to weak by slightly too weak.

C10-T11. Exercise 10.13: missing 0 in the pole set

- **Precise location:** printed p. 284, Exercise 10.13; the exact printed wording or display is reproduced or quoted under **Printed text**.
- **Printed text:** "except on $\{z =\} = \{u = -\infty\}$".
- **Correction / repair:** Replace by "except on $\{z = 0\} = \{u = -\infty\}$".

C10-T14. Exercise 10.20: final clause can be made grammatical

- **Precise location:** printed p. 284, Exercise 10.20; the exact printed wording or display is reproduced or quoted under **Printed text**.
- **Printed text:** "uniform with respect to ψ if ψ stays in a compact set $\mathcal{E}^1(X, \omega)$".
- **Correction / repair:** Replace by "uniformly for ψ ranging over a compact subset of $\mathcal{E}^1(X, \omega)$".

Chapter 11. The variational approach

Range checked: printed pp. 289–314.

Mathematical corrections and proof clarifications

C11-M02. Theorem 11.7: the auxiliary function $g(t)$ is not defined for $\lambda = 0$

- **Precise location:** printed p. 294, Theorem 11.7; the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** The proof defines
$$g(t) = E(P(\varphi + tv)) + \frac{1}{\lambda} \log \int_X e^{-\lambda(\varphi+tv)}\, d\mu.$$
This formula is undefined in the case $\lambda = 0$, but Theorem 11.7 includes $\lambda = 0$.
- **Correction / repair:** Add the separate definition
$$g(t) = E(P(\varphi + tv)) - \int_X (\varphi + tv)\, d\mu$$
when $\lambda = 0$, and state that the derivative formula becomes
$$g'(0) = \int_X v\, MA(\varphi) - \int_X v\, d\mu.$$

C11-M08. Theorem 11.13: $\alpha_\mu(X)$ is undefined

- **Precise location:** printed p. 300, Theorem 11.13; the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** Definition 11.12 defines
$$\alpha_\mu(\{\omega\}),$$
and in the Fano case $\alpha(X) := \alpha_\mu(c_1(X))$. Theorem 11.13 states the hypothesis as
$$\alpha_\mu(X) > \lambda n/(n+1),$$
but $\alpha_\mu(X)$ has not been defined.
- **Correction / repair:** Replace $\alpha_\mu(X)$ by $\alpha_\mu(\{\omega\})$, or, if the intended setting is Fano, explicitly say $\{\omega\} = c_1(X)$ and use $\alpha(X)$.

C11-M09. Theorem 11.13 proof: a critical point gives the normalized equation first

- **Precise location:** printed p. 300, Theorem 11.13 proof; the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** The proof ends by saying that a maximizer of F_1 is a critical point, "hence a desired solution." By Lemma 11.2, the critical point first gives
$$MA(\varphi) = \frac{e^{-\varphi}\mu}{\int_X e^{-\varphi}\,d\mu}.$$
To solve the exact equation $MA(\tilde{\varphi}) = e^{-\tilde{\varphi}}\mu$, one must add the constant
$$\tilde{\varphi} = \varphi + \log \int_X e^{-\varphi}\,d\mu.$$
- **Correction / repair:** Add this final normalization sentence, as in Lemma 11.2.

C11-M17. Proposition 11.24 proof: the integrability space for g is wrong

- **Precise location:** printed p. 309, Proposition 11.24 proof; the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** The proof writes that one can represent
$$\mu = g(\omega + dd^c u)^n$$
with u bounded and $0 \le g \in L^1(\mu)$. The natural and sufficient condition is
$$g \in L^1((\omega + dd^c u)^n),$$
since $\int g(\omega + dd^c u)^n = \mu(X)$. Requiring $g \in L^1(\mu)$ would mean $g^2 \in L^1((\omega + dd^c u)^n)$, which is stronger and not guaranteed.
- **Correction / repair:** Replace $g \in L^1(\mu)$ by $g\,d(\omega + dd^c u)^n \in L^1$, or simply $g \in L^1((\omega + dd^c u)^n)$.

C11-M18. Proposition 11.24 proof: the truncation indicator is reversed

- **Precise location:** printed p. 310, Proposition 11.24 proof; the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** For $\varphi_i^{(k)} = \max(\varphi_i, -k)$, the proof defines
$$f_i^{(k)} = \mathbf{1}_{\{\varphi_i < -k\}} f_i.$$

This is the wrong set. On $\{\varphi_i > -k\}$, the truncation agrees with φ_i, and the maximum principle gives

$$(\omega + dd^c\varphi_i^{(k)})^n \geq \mathbf{1}_{\{\varphi_i>-k\}}(\omega + dd^c\varphi_i)^n \geq \mathbf{1}_{\{\varphi_i>-k\}} f_i\mu.$$

With the printed indicator $\mathbf{1}_{\{\varphi_i<-k\}}$, the density tends to zero rather than to f_i, except on the pluripolar pole set.

- **Correction / repair:** Replace

$$\mathbf{1}_{\{\varphi_i<-k\}} f_i$$

by

$$\mathbf{1}_{\{\varphi_i>-k\}} f_i$$

(or $\mathbf{1}_{\{\varphi_i\geq-k\}} f_i$, the boundary being harmless after the usual convention/approximation).

C11-M14. Definition 11.19: the supporting exercise is cited incorrectly

- **Type:** cross-reference error.
- **Precise location:** printed p. 304, discussion immediately following Definition 11.19, citation to Exercise 10.18.
- **Correction / repair:** replace "Exercise 10.18" by "Exercise 10.16," which is the exercise establishing the asserted property of the energy class.

Typographical and editorial corrections

C11-T05. Definition 11.19: repair the relative clause defining the power weight

- **Precise location:** printed p. 304, paragraph after Definition 11.19, sentence containing the power weight.
- **Printed text / issue:** "for the weight $\chi : t \mapsto -(-t)^p$ lies in $\mathcal{W}$" has no grammatical attachment.
- **Correction / repair:** write "defined using the weight $\chi(t) = -(-t)^p$, which lies in $\mathcal{W}$."

C11-T01. Proposition 11.3 proof: wrong sign in the list of cases

- **Precise location:** printed p. 291, Proposition 11.3 proof; the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** "for each case $\lambda > 0$, $\lambda = 0$, $\lambda > 0$".
- **Correction / repair:** "for each case $\lambda < 0$, $\lambda = 0$, $\lambda > 0$".

C11-T02. Corollary 11.9 proof: number agreement

- **Precise location:** printed p. 296, Corollary 11.9 proof; the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** "if ψ is another solutions".
- **Correction / repair:** "if ψ is another solution".

C11-T03. Section 11.3.1: extra space before parenthesis

- **Precise location:** printed p. 299, Section 11.3.1; the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** "when the first Chern class $c_1(X)$ of X is positive)".
- **Correction / repair:** "when the first Chern class $c_1(X)$ of X is positive)".

C11-T06. Section 11.4: misspelling

- **Precise location:** printed p. 305, Section 11.4; the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** “adressed”.
- **Correction / repair:** “addressed”.

C11-T07. Theorem 11.23 proof: misspelling

- **Precise location:** printed p. 308, Theorem 11.23 proof; the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** “pertubing”.
- **Correction / repair:** “perturbing”.

C11-T08. Proposition 11.24 proof: background omitted in PSH

- **Precise location:** printed p. 309, Proposition 11.24 proof; the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** The display says $u \in PSH(X) \cap L^\infty(X)$.
- **Correction / repair:** $u \in PSH(X, \omega) \cap L^\infty(X)$.

C11-T09. Lemma 11.25 proof: singular/plural disagreement

- **Precise location:** printed p. 311, Lemma 11.25 proof; the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** “local psh function that are bounded”.
- **Correction / repair:** “local psh functions that are bounded”.

C11-T10. Lemma 11.25 proof: preposition

- **Precise location:** printed p. 311, Lemma 11.25 proof; the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** “puts no mass to the complement”.
- **Correction / repair:** “puts no mass on the complement”.

C11-T11. Exercise 11.8: agreement for sequences

- **Precise location:** printed p. 313, Exercise 11.8; the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** “one can choose ε_j (resp. δ_j) which decreases slowly (resp. fast)”.
- **Correction / repair:** “which decrease slowly (resp. fast)”.

Chapter 12. Uniform a priori estimates

Range checked: printed pp. 315–342.

Mathematical corrections and proof clarifications

C12-M01. Corollary 12.4: monotonicity of normalized capacities is used as if capacities were unnormalized

- **Precise location:** printed p. 317, proof of Corollary 12.4; the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** The proof uses the implication $\omega_1 \le \omega_2 \Rightarrow \mathrm{Cap}_{\omega_1} \le \mathrm{Cap}_{\omega_2}$. With the normalized convention used earlier,
$$MA_\omega(u) = V_\omega^{-1}(\omega + dd^c u)^n,$$
this is not literally true: both the admissible class and the normalizing volume vary. The printed chain involving the Alexander-Taylor comparison should also keep the order of T_{ω_1} and T_{ω_2} consistent.
- **Correction / repair:** Either switch explicitly to unnormalized capacities in this proof, or insert the volume/scaling factors. For the normalized capacity, even the simple scaling $\omega_2 = \lambda\omega_1$ produces powers of λ, not equality-level monotonicity.

C12-M02. Theorem 12.5: the coefficient in the integral defining φ_ε is wrong

- **Precise location:** printed p. 318, proof of Josefson's theorem; the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** The proof defines
$$\varphi_\varepsilon(x) = \frac{1}{\varepsilon}\int_1^{+\infty} \frac{V_t(x) - \sup_X V_t}{t^{1+\varepsilon}}\,dt$$
and then claims that this is $A\omega$-psh with
$$A = \varepsilon^{-1}\int_1^{+\infty} t^{-(1+\varepsilon)}dt = 1.$$
The displayed integral is actually ε^{-2}, not 1.
- **Correction / repair:** Replace the prefactor $1/\varepsilon$ by ε, or keep the prefactor and then divide the resulting function by $A = \varepsilon^{-2}$. Adjust the later constants accordingly.

C12-M04. Proposition 12.6: the Skoda constant has the wrong dependence on p

- **Precise location:** printed p. 319, Proposition 12.6; the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** The printed constant is of the form
$$C = (p-1)^{-2n}A\|f\|_p.$$
The proof uses Holder with the conjugate exponent $q = p/(p-1)$, and Skoda-type integrability gives powers of q. The constant should involve q^{2n}, i.e.
$$\left(\frac{p}{p-1}\right)^{2n},$$
up to the universal Skoda constant, rather than only $(p-1)^{-2n}$.
- **Correction / repair:** Replace the constant by $C = Aq^{2n}\|f\|_{L^p}$, or by an equivalent expression with $q = p/(p-1)$.

C12-M07. Theorem 12.1 / Lemma 12.8: the smallness threshold for $g(t_0)$ is not the one needed by the lemma

- **Precise location:** printed pp. 320–321, Theorem 12.1 / Lemma 12.8; the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** Lemma 12.8 requires a smallness condition of the type $g(t_0) < e^{-1}C^{-1/n}$, but the proof of Theorem 12.1 uses the weaker threshold $g(t_0) < 1/(2C^{1/n})$. The subsequent choice of t_0 has the corresponding factor problem; from the displayed estimate one needs at least

$$t_0 > 1 + 2^n C C_2 \|f\|_{L^p(dV)},$$

  and for the stated Lemma 12.8 threshold one should use an e^n-type constant.
- **Correction / repair:** Align the threshold in Theorem 12.1 with Lemma 12.8, and replace the printed factor 2^{n-1} by the correct larger constant.

C12-M08. Lemma 12.8: endpoint/infimum convention should be made strict

- **Precise location:** printed p. 321, Lemma 12.8; the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** The proof uses an infimum argument for the first time where the function could cross a threshold. To avoid endpoint ambiguity, one should either use strict inequalities throughout or add a right-continuity argument.
- **Correction / repair:** Replace the infimum by $s_0 + \eta$, prove the estimate there, and let $\eta \downarrow 0$, or state the needed right-continuity of g.

C12-M10. Proposition 12.10: the substitutions after Lemma 12.9 are not literally correct

- **Precise location:** printed p. 322, Proposition 12.10; the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** The proof says that (12.2.1) follows by the substitutions

$$t \mapsto t - M\delta, \qquad \delta \mapsto (1+M)\delta.$$

  With $-M \le \psi \le 0$, these substitutions do not give the displayed sublevel set with the right inclusion direction.
- **Correction / repair:** Apply Lemma 12.9 with

$$\eta = \frac{\delta}{1+M}, \qquad T = t + M\eta.$$

  Then

$$\eta^n \operatorname{Cap}_\omega\{\varphi - \psi < -t - \delta\} \le \int_{\{\varphi-\psi<-t\}} MA(\varphi),$$

  and multiplying by $(1+M)^n$ gives the desired estimate.

C12-M11. Lemma 12.15: the comparison misses a normalization factor

- **Precise location:** printed p. 325, Lemma 12.15; the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** The proof compares unnormalized integrals involving $(\omega/2 + dd^c u)^n$, but the capacity $\operatorname{Cap}_{\omega/2}$ is normalized by the volume of $\omega/2$, namely $2^{-n}V_\omega$. Thus the printed inequality

$$\operatorname{Cap}_{\omega/2}(E) \le \operatorname{Cap}_\psi(E)$$

  misses a factor.

- **Correction / repair:** Under normalized capacities, write

$$\mathrm{Cap}_{\omega/2}(E) \leq 2^n \,\mathrm{Cap}_{\psi}(E),$$

or use unnormalized capacities in this local argument.

C12-M14. Proposition 12.17: the common dominating measure omits μ_1

- **Precise location:** printed p. 328, Proposition 12.17; the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** The proof defines a common measure of the form

$$\nu = 2^{-1}\mu + \sum_{j\geq 2} 2^{-j}\mu_j.$$

This omits μ_1. If μ_1 is not absolutely continuous with respect to the displayed ν, the later writing $\mu_j = f_j\nu$ for all j is not justified.

- **Correction / repair:** Use

$$\nu = 2^{-1}\mu + \sum_{j\geq 1} 2^{-j-1}\mu_j,$$

or first discard finitely many indices and state that explicitly.

C12-M19. Theorem 12.21: the proof changes the reference measure from dV to ω^n

- **Precise location:** printed p. 330, Theorem 12.21 statement and definition of $I(\varphi, \psi)$; printed p. 331, first Cauchy–Schwarz display beginning $I(\varphi, \psi) =$.
- **Issue:** the equations and normalization use densities relative to the fixed volume form dV, but the proof later writes $I = \int(\varphi - \psi)(g - f)\,\omega^n$. This is not the same measure unless $dV = \omega^n$, and normalized MA also contributes the fixed volume factor.
- **Correction / repair:** retain dV in the Cauchy–Schwarz step (with the fixed factor relating MA to ω_φ^n), or state before the theorem that $dV = \omega^n$. The former preserves the theorem for an arbitrary smooth reference volume.

C12-M20. Theorem 12.21: a wedge factor is misprinted as ω_{φ_1}

- **Precise location:** printed p. 330, integration-by-parts line, Theorem 12.21; the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** One line contains a factor ω_{φ_1}, but there is no φ_1 in this theorem. The factor should be ω_φ or the appropriate term in the mixed product between ω_φ and ω_ψ.
- **Correction / repair:** Replace ω_{φ_1} by ω_φ.

C12-M21. Proposition 12.22: the statement is missing the exponent n on ω_ψ

- **Precise location:** printed p. 332, Proposition 12.22; the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** The proposition assumes

$$\omega_\varphi^n = f\omega^n, \qquad \omega_\psi = g\omega^n.$$

The second equation should be

$$\omega_\psi^n = g\omega^n.$$

- **Correction / repair:** Insert the missing exponent n.

C12-M26. Theorem 12.23: the minimizer step in the Kiselman-Legendre transform has a missing logarithmic term

- **Precise location:** printed p. 334, Theorem 12.23; the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** If $t(z)$ realizes the infimum in
$$\psi_{c,\delta} = \rho_t\varphi + Kt^2 - c\log(t/\delta),$$
then
$$\rho_{t(z)}\varphi + Kt(z)^2 - \varphi = \psi_{c,\delta} - \varphi + c\log(t(z)/\delta),$$
not simply $\psi_{c,\delta} - \varphi$. Since $t(z) \le \delta$, the missing logarithmic term is non-positive.
- **Correction / repair:** Replace the printed equality by the corresponding inequality
$$\rho_{t(z)}\varphi + Kt(z)^2 - \varphi \le \psi_{c,\delta} - \varphi.$$

C12-M27. Theorem 12.23: formula (12.4.4) is missing the factor δ^α

- **Precise location:** printed p. 335, formula (12.4.4); the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** From (12.4.3), after replacing δ by $\kappa(z)^{-1}\delta$, one obtains a bound containing δ^α. The displayed formula (12.4.4) drops this factor and becomes independent of δ, which cannot yield Holder continuity.
- **Correction / repair:** Replace (12.4.4) by a bound of the form
$$\rho_\delta\varphi(z) - \varphi(z) \le C_3\delta^\alpha \exp\{2\alpha C_2(C_1 - \psi_0(z))\}.$$

C12-M31. Proposition 12.30: small ordinary capacity is not shown to imply small relative capacity

- **Precise location:** printed p. 338, proof of the domination principle; the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** The proof chooses a continuous approximation $\tilde{u}$ with
$$\mathrm{Cap}_\omega(\{u \ne \tilde{u}\}) < \varepsilon.$$
It later needs to control
$$\mathrm{Cap}_{u,0}(\{u \ne \tilde{u}\}).$$
Theorem 12.27 only says that pluripolar sets have zero relative outer capacity; it does not imply that an arbitrary small ordinary-capacity open exceptional set has small $(u, 0)$-capacity.
- **Correction / repair:** Insert a comparison estimate between $\mathrm{Cap}_{u,0}$ and Cap_ω for the chosen exceptional sets, or choose a decreasing exceptional sequence whose intersection is pluripolar and prove relative capacity continuity along it.

C12-M32. Lemma 12.31: the lower obstacle φ is missing from the hypotheses

- **Precise location:** printed p. 338, Lemma 12.31; the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** The lemma refers to the "$(\varphi, 0)$-extremal function" but the statement does not fix a lower obstacle φ with the required properties.
- **Correction / repair:** Begin the statement with: "Let $\varphi \in \mathcal{E}(X, \omega)$, $\varphi \le 0$, and let $v \in PSH(X, \omega)$, $v \le 0$." Adjust the exact energy hypothesis to what is used.

C12-M35a. Exercise 12.5: the Orlicz integral is taken over the wrong space

- **Precise location:** printed p. 339, Exercise 12.5, display after the words *the density f belongs to the Orlicz class*.
- **Printed text / issue:** f is a density on X, but the display integrates $\chi \circ f(t)$ over $[0, +\infty)$.
- **Correction / repair:** replace it by $\int_X \chi(f)\, dV < +\infty$.

C12-M35b. Exercise 12.10(2): the boundary datum is not well defined on the circle

- **Precise location:** printed p. 340, Exercise 12.10(2), boundary prescription $\Phi(e^{2\pi i\theta}) = |\sin\theta|$.
- **Issue:** the right side is not 1-periodic in θ, so it does not define a function of $e^{2\pi i\theta}$.
- **Correction / repair:** use $\Phi(e^{i\theta}) = |\sin\theta|$, or $\Phi(e^{2\pi i\theta}) = |\sin(\pi\theta)|$.

C12-M35c. Exercise 12.14(iii): the model density is not a Radon density

- **Precise location:** printed pp. 341–342, opening hypothesis of Exercise 12.14 and part (iii), the model $f \asymp |s_D|^{-2}$.
- **Issue:** $|s_D|^{-2}$ is not locally integrable in a transverse complex coordinate, contradicting the opening assumption that $f\omega^n$ is a Radon measure.
- **Correction / repair:** either work with a σ-finite measure locally finite on $X \setminus D$ and separately require $e^{\varphi} f$ to be integrable, or retain the Radon hypothesis and use an integrable model such as $|s_D|^{-2(1-\varepsilon)}$, $\varepsilon > 0$.

Typographical and editorial corrections

C12-T01. “Kolodiej’s”

- **Precise location:** printed p. 315; the exact printed wording or display is reproduced or quoted under **Correction / repair**.
- **Correction / repair:** “Kolodiej’s idea” $\longrightarrow$ “Kolodziej’s idea” or “Kołodziej’s idea”.

C12-T02. “lectures notes”

- **Precise location:** printed p. 315; the exact printed wording or display is reproduced or quoted under **Correction / repair**.
- **Correction / repair:** “some more recent lectures notes” $\longrightarrow$ “some more recent lecture notes”.

C12-T03. “its L^q-norms is”

- **Precise location:** printed p. 320; the exact printed wording or display is reproduced or quoted under **Correction / repair**.
- **Correction / repair:** “its $L^q(dV)$-norms is bounded” $\longrightarrow$ “its $L^q(dV)$-norm is bounded” or “its $L^q(dV)$-norms are bounded”.

C12-T04. Duplicate “that”

- **Precise location:** printed p. 322, proof of Lemma 12.9; the exact printed wording or display is reproduced or quoted under **Correction / repair**.
- **Correction / repair:** “It follows from the comparison principle that that” $\longrightarrow$ “It follows from the comparison principle that”.
- **Same printed page, later in the proof:** “interchange the roles of φ and and get” $\longrightarrow$ “interchange the roles of φ and get”.

C12-T05. "We an assume"

- **Precise location:** printed p. 323, proof of Theorem 12.11; the exact printed wording or display is reproduced or quoted under **Correction / repair**.
- **Correction / repair:** "We an assume without loss of generality" $\longrightarrow$ "We can assume without loss of generality".

C12-T06. "If follows that"

- **Precise location:** printed p. 330, proof of Lemma 12.19; the exact printed wording or display is reproduced or quoted under **Correction / repair**.
- **Correction / repair:** "If follows that" $\longrightarrow$ "It follows that".

C12-T07. Remark 12.20: "usc condition" should be "lsc condition"

- **Precise location:** printed p. 330, Remark 12.20; the exact printed wording or display is reproduced or quoted under **Correction / repair**.
- **Correction / repair:** The remark discusses lower semi-continuity of $\varphi \mapsto \int_X \varphi\, d\mu$, and says upper semi-continuity is automatic by Fatou. The next sentence should say that the **lsc** condition is not automatic.

C12-T08. "coholomogy"

- **Precise location:** printed p. 333, proof of Corollary 12.25; the exact printed wording or display is reproduced or quoted under **Correction / repair**.
- **Correction / repair:** "coholomogy class" $\longrightarrow$ "cohomology class".

C12-T09. "They allow to give"

- **Precise location:** printed p. 335; the exact printed wording or display is reproduced or quoted under **Correction / repair**.
- **Correction / repair:** "They allow to give a simple proof" $\longrightarrow$ "They allow one to give a simple proof".

C12-T10. "requires to analyze"

- **Precise location:** printed p. 336; the exact printed wording or display is reproduced or quoted under **Correction / repair**.
- **Correction / repair:** "requires to analyze" $\longrightarrow$ "requires analyzing".

C12-T12. Exercise 12.4: missing lower limit and variable in an integral

- **Precise location:** printed p. 339, Exercise 12.4; the exact printed wording or display is reproduced or quoted under **Correction / repair**.
- **Correction / repair:** Replace "such that $\int^{+\infty} \varepsilon < +\infty$" by something like

$$\int_0^{+\infty} \varepsilon(t)\, dt < +\infty.$$

Chapter 13. The viscosity approach on compact manifolds

Range checked: printed pp. 343–362.

Mathematical corrections and proof clarifications

C13-M02. Definition 13.7: wrong codomain for supersolutions

- **Precise location:** printed p. 345, Definition 13.7; the exact printed wording or display is reproduced or quoted under **Printed text**.
- **Printed text:** $\varphi : X \to \mathbb{R} \cup \{-\infty\}$ is lower semicontinuous, bounded from below, and $\varphi \not\equiv +\infty$.
- **Issue:** The codomain and nontriviality condition are inconsistent. A lower semicontinuous supersolution may take $+\infty$, not $-\infty$.
- **Correction / repair:** Write
$$\varphi : X \to \mathbb{R} \cup \{+\infty\}, \qquad \varphi \not\equiv +\infty.$$
If only finite supersolutions are intended, use $\varphi : X \to \mathbb{R}$ and delete the nontriviality clause.

C13-M06. Lemma 13.14: the gradient of the quadratic penalty has the wrong factor and sign relation

- **Precise location:** printed p. 348, Lemma 13.14; the exact printed wording or display is reproduced or quoted under **Printed text**.
- **Printed text:** For $\phi_j(x, y) = |x - y|^2/(2\varepsilon_j)$, the text writes
$$p^+ = \frac{x_j - y_j}{2\varepsilon_j} = -p_-.$$
- **Issue:** Differentiation gives
$$D_x\phi_j(x_j, y_j) = \frac{x_j - y_j}{\varepsilon_j}, \qquad -D_y\phi_j(x_j, y_j) = \frac{x_j - y_j}{\varepsilon_j}.$$
- **Correction / repair:** Replace the printed chain by
$$p^+ = \frac{x_j - y_j}{\varepsilon_j} = p_-.$$
The later determinant comparison is unaffected because the equation has no gradient dependence.

C13-M07. Lemma 13.14: the Hessian matrix A is written with the wrong parameter

- **Precise location:** printed p. 348, Lemma 13.14; the exact printed wording or display is reproduced or quoted under **Printed text**.
- **Printed text:** After defining $A = D^2\phi_j(x_j, y_j)$, the text writes
$$A = \gamma^{-1}\begin{pmatrix} I & -I \\ -I & I \end{pmatrix}.$$
- **Issue:** Since $\phi_j(x, y) = |x - y|^2/(2\varepsilon_j)$, its Hessian uses the penalty scale ε_j, whereas γ is the independent parameter in the Jensen–Ishii matrix inequality.
- **Correction / repair:** Replace the display by
$$A = \varepsilon_j^{-1}\begin{pmatrix} I & -I \\ -I & I \end{pmatrix}.$$
The later choice $\gamma = \varepsilon_j$ then gives the stated bound.

C13-M08. Theorem 13.16: Exercise 13.3 applies to the regularized Perron envelope

- **Precise location:** printed p. 351, proof of Theorem 13.16, first sentence after the definition of the raw supremum φ.
- **Issue:** the sentence says that the supremum φ is a subsolution by Exercise 13.3, but that exercise proves the assertion for the upper-semicontinuous regularization φ^*.
- **Correction / repair:** write φ^* *is a viscosity subsolution by Exercise 13.3.* Keep φ as the raw supremum; the later comparison proves $\varphi = \varphi^* = \varphi_*$ and hence supplies continuity.

C13-M09. Theorem 13.16 proof: $F_+(q) < 0$ does not force $v_{x_0} > 0$

- **Precise location:** printed p. 351, proof of Theorem 13.16; the exact printed wording or display is reproduced or quoted under **Printed text**.
- **Printed text:** From $F_+(q^{(2)}_{x_0}) < 0$, the proof says that $v(x_0) > 0$, and then assumes $v > 0$ on a small ball.
- **Issue:** If $v(x_0) = 0$ but $\omega + dd^c q$ is positive definite at x_0, then

  $$F_+(q^{(2)}_{x_0}) = -\det(\omega + dd^c q)_{x_0} < 0.$$

  Thus $F_+ < 0$ does not imply $v(x_0) > 0$.
- **Correction / repair:** do not infer $v(x_0) > 0$. The inequality $F_+(q^{(2)}_{x_0}) < 0$ forces positive definiteness of the complex Hessian; by continuity, $F = F_+ < 0$ on a small neighborhood. Run the local bump argument there. Positivity of v is neither available nor needed.

C13-M14. Exercise 13.5: the maximizing points are not defined

- **Precise location:** printed p. 361, Exercise 13.5; the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** The exercise asks to show

$$\alpha d^2(x_\alpha, y_\alpha) \to 0$$

but the exercise never defines (x_α, y_α).

- **Correction / repair:** Add: "Let $(x_\alpha, y_\alpha) \in X \times X$ be a point at which the supremum defining M_α is attained." Alternatively, choose approximate maximizers and include the approximation error in the conclusion.

C13-M15. Exercise 13.6: missing mass normalization for f

- **Precise location:** printed p. 361, Exercise 13.6; the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** The exercise assumes only $f \geq 0$ continuous and asks for uniform convergence, as $\varepsilon \downarrow 0$, to the solution of

  $$(\omega + dd^c\psi)^n = f\omega^n.$$

  This limiting equation is solvable only if

  $$\int_X f\,\omega^n = \int_X \omega^n.$$

  If this condition fails, solutions of

  $$(\omega + dd^c\varphi_\varepsilon)^n = e^{\varepsilon\varphi_\varepsilon} f\omega^n$$

  typically drift by a constant of order $1/\varepsilon$, rather than converging uniformly to a finite limit.

- **Correction / repair:** Add the mass normalization $\int_X f\,\omega^n = \int_X \omega^n$ and assume that f is not identically zero. The condition
$$\int_X \psi f \omega^n = 0$$
can then select the additive constant of the $\varepsilon = 0$ solution.

Typographical and editorial corrections

C13-T01. "theses authors"

- **Precise location:** printed p. 343, introduction; the exact printed wording or display is reproduced or quoted under **Printed text**.
- **Printed text:** allowed theses authors
- **Correction / repair:** allowed these authors.

C13-T02. "vicosity"

- **Precise location:** printed p. 345, Corollary 13.6; the exact printed wording or display is reproduced or quoted under **Printed text**.
- **Printed text:** "in the vicosity sense".
- **Correction / repair:** "in the viscosity sense".

C13-T03. "a honest"

- **Precise location:** printed p. 346, beginning of Section 13.2.2; the exact printed wording or display is reproduced or quoted under **Printed text**.
- **Printed text:** a honest global comparison principle
- **Correction / repair:** an honest global comparison principle.

C13-T04. "enable us"

- **Precise location:** printed p. 353, proof of Corollary 13.19; the exact printed wording or display is reproduced or quoted under **Printed text**.
- **Printed text:** "Theorem 13.16 enable us to conclude".
- **Correction / repair:** "Theorem 13.16 enables us to conclude".

C13-T05. "by the the family"

- **Precise location:** printed p. 357, proof of Theorem 13.22; the exact printed wording or display is reproduced or quoted under **Printed text**.
- **Printed text:** reached by the the family
- **Correction / repair:** reached by the family.

C13-T06. Missing "be" in Lemma 13.24

- **Precise location:** printed p. 358, Lemma 13.24; the exact printed wording or display is reproduced or quoted under **Printed text**.
- **Printed text:** "Let φ a bounded plurisubharmonic function".
- **Correction / repair:** "Let φ be a bounded plurisubharmonic function".

C13-T07. "satisifies"

- **Precise location:** printed p. 360, proof of Theorem 13.25; the exact printed wording or display is reproduced or quoted under **Printed text**.
- **Printed text:** "Hence u_ϵ satisifies".
- **Correction / repair:** "Hence u_ϵ satisfies".

Chapter 14. Smooth solutions

Range checked: printed pp. 363–386.

Mathematical corrections and proof clarifications

C14-M01. Proposition 14.2: the Monge–Ampère linearization is missing its density factor

- **Precise location:** printed p. 364, definition of the density-valued map $\mathcal{M} : A \to B$; printed p. 365, first derivative display and the following assertion that $\Delta_\varphi : E \to F$ is an isomorphism.
- **Issue:** for $\mathcal{M}(u) = \omega_u^n/\omega^n$,
$$D\mathcal{M}(\varphi) \cdot v = \frac{\omega_\varphi^n}{\omega^n}\,\Delta_\varphi v,$$
not $\Delta_\varphi v$. The unweighted Laplacian has zero mean with respect to ω_φ^n, not the fixed ω^n defining the tangent space F.
- **Correction / repair:** use the displayed weighted Laplacian as the differential $E \to F$. It has zero ω^n-mean and is an isomorphism after quotienting/fixing constants, as required by the inverse-function theorem.

C14-M04. Theorem 14.3: the proof treats the auxiliary volume form as ω^n

- **Precise location:** printed p. 366, Theorem 14.3, equation with arbitrary smooth dV and the stated dependence of A; printed p. 370, opening Ricci identity in the proof of Lemma 14.7.
- **Issue:** from $\omega_\varphi^n = e^{\psi^+ - \psi^-}\,dV$, the Ricci identity contains the curvature of dV. The proof instead writes the identity with $\mathrm{Ric}(\omega)$, which is valid only when $dV = \omega^n$.
- **Correction / repair:** either set $dV = \omega^n$ in the theorem, or replace the background Ricci term by $\mathrm{Ric}(dV) := -dd^c \log dV$. Then let A depend on an upper ω-bound for $\mathrm{Ric}(dV)$ and on the comparison constants between dV and ω^n.

C14-M07. Proposition 14.12: the uniform Schauder constant also needs a positive lower bound for f

- **Precise location:** printed p. 377, Proposition 14.12; the exact printed wording or display is reproduced or quoted under **Printed text**.
- **Printed text:** the constant depends only on upper bounds for $\|\varphi\|_{2,\alpha}$ and $\|f\|_{k,\alpha}$.
- **Issue:** Uniform ellipticity of the linearized Monge–Ampère operator also requires a lower bound for the eigenvalues of $\omega + dd^c\varphi$. Given a C^2 upper bound, this is obtained from a positive lower bound for f; it is not controlled by an upper $C^{k,\alpha}$ norm alone.
- **Correction / repair:** Add a positive lower bound $f \geq c_0 > 0$ (equivalently, a bound for $\|f^{-1}\|_{C^0}$) to the data on which the Schauder constant depends.

C14-M08. Theorem 14.1: the approximation argument lacks uniform right-hand-side regularity

- **Precise location:** printed p. 378, proof of Theorem 14.1, paragraph beginning *We let φ_j denote a sequence of smooth ω-psh functions*, and the final two sentences invoking Theorem 14.9 and Proposition 14.12.

- **Issue:** uniform convergence $\varphi_j \to \varphi$ gives no uniform Lipschitz bound for $F(\varphi_j) + h + c_j$, although Theorem 14.9 depends on precisely such a bound. Even if a uniform $C^{2,\alpha}$ estimate for the solutions ψ_j were obtained, Proposition 14.12 cannot bootstrap their equations from arbitrary φ_j whose higher derivatives remain uncontrolled.

- **Repair:** use the Chapter 12 Hölder estimate to choose regularizations with a uniform C^γ modulus and apply the standard complex Evans–Krylov estimate with C^γ right-hand side. This gives the limit $\varphi \in C^{2,\alpha}$. Then bootstrap the *limit equation* $\omega_\varphi^n = e^{F(\varphi)+h}\omega^n$ directly; do not claim uniform higher derivatives of the auxiliary ψ_j from uncontrolled derivatives of φ_j.

C14-M11. Theorem 14.16 proof: the key maximum-principle inequality contains an unjustified extra ψ^- term

- **Precise location:** printed p. 382, proof of Theorem 14.16; the exact printed wording or display is reproduced or quoted under **Printed text**.

- **Printed text:**

$$H(x) \leq H(x_0) \leq A_2 + \psi^-(x_0) - (3C+1)(\varphi - \phi)(x_0).$$

- **Issue:** The preceding estimate gives

$$\log \operatorname{tr}_\omega(\omega_\varphi)(x_0) \leq A_2 - \psi^-(x_0).$$

  Since

$$H = \log \operatorname{tr}_\omega(\omega_\varphi) + \psi^- - (3C+1)(\varphi - \phi),$$

  this yields

$$H(x_0) \leq A_2 - (3C+1)(\varphi - \phi)(x_0),$$

  not the printed inequality with $+\psi^-(x_0)$. The printed inequality would require an additional lower bound on ψ^- at x_0, which is not part of the hypotheses.

- **Correction / repair:** Delete $+\psi^-(x_0)$ from the displayed upper bound for $H(x_0)$, so that it reads

$$H(x_0) \leq A_2 - (3C+1)(\varphi - \phi)(x_0).$$

C14-M12. Theorem 14.16 and Exercise 14.16 do not match for general λ

- **Precise location:** printed p. 381, proof of Theorem 14.16, sentence referring the case $\lambda > 0$ to Exercise 14.16; and printed p. 386, Exercise 14.16, displayed equation.

- **Issue:** Theorem 14.16 is stated for arbitrary $\lambda \geq 0$, and the proof says the case $\lambda > 0$ is left to Exercise 14.16. But Exercise 14.16 treats only the equation

$$(\theta + t\omega + dd^c\varphi)^n = e^{\varphi + \psi^+ - \psi^-}\omega^n,$$

  i.e. the case $\lambda = 1$, not arbitrary $\lambda > 0$.

- **Correction / repair:** State Exercise 14.16 for

$$(\theta + t\omega + dd^c\varphi)^n = e^{\lambda\varphi+\psi^+-\psi^-}\omega^n, \qquad \lambda > 0,$$

and track λ in the maximum-principle coefficient. If the exercise is left unchanged, restrict the deferred case in Theorem 14.16 to $\lambda = 1$.

C14-M15. Exercise 14.1: Rayleigh quotient for the first non-zero eigenvalue lacks the orthogonality condition

- **Precise location:** printed p. 383, Exercise 14.1; the exact printed wording or display is reproduced or quoted under **Printed text**.
- **Printed text:**

$$\lambda_1 := \inf \frac{\|\nabla_\omega h\|^2_{L^2}}{\|h\|^2_{L^2}},$$
$$h \in C^\infty(X, \mathbb{R}).$$

- **Issue:** Without excluding constants, the infimum is 0, not the first non-zero eigenvalue.
- **Correction / repair:** Require

$$\int_X h\,\omega^n = 0,$$
$$h \not\equiv 0.$$

C14-M16. Exercise 14.2: local Sobolev spaces only give local Hölder/Lipschitz conclusions

- **Precise location:** printed p. 383, Exercise 14.2; the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** From $W^{1,p}_{loc}(\Omega)$ one obtains $C^{0,1-k/p}_{loc}$ for $p > k$, not a global $C^{0,1-k/p}(\Omega)$ conclusion. Similarly $W^{1,\infty}_{loc}$ corresponds to locally Lipschitz functions, not necessarily globally Lipschitz functions on all of Ω.
- **Correction / repair:** Replace $C^{0,1-k/p}(\Omega)$ by $C^{0,1-k/p}_{loc}(\Omega)$ and "Lipschitz" by "locally Lipschitz".

C14-M17. Exercise 14.4: the elliptic L^p range and the definition of $W^{2,p}_{loc}$ are wrong

- **Precise location:** printed p. 383, Exercise 14.4; the exact printed wording or display is reproduced or quoted under **Printed text**.
- **Printed text:**

$$\Delta u \in L^p_{loc} \Rightarrow u \in W^{2,p}_{loc} \quad \text{for all } 1 \le p < +\infty,$$

and

$$W^{2,p}_{loc} = \{f \mid D^\alpha f \in L^2_{loc} \text{ for } |\alpha| \le 2\}.$$

- **Issue:** Standard Calderón–Zygmund $W^{2,p}$ estimates hold for $1 < p < \infty$, not at the endpoint $p = 1$ in this strong form. Also the definition of $W^{2,p}_{loc}$ should use L^p_{loc}, not L^2_{loc}.
- **Correction / repair:** Use $1 < p < +\infty$ and define

$$W^{2,p}_{loc} = \{f \mid D^\alpha f \in L^p_{loc} \text{ for all } |\alpha| \le 2\}.$$

C14-M19. Exercise 14.14: the Hölder exponent is written with an undefined α

- **Precise location:** printed p. 386, Exercise 14.14; the exact printed wording or display is reproduced or quoted under **Printed text**.
- **Printed text:** u is Hölder continuous of exponent $\min(\alpha, \gamma^-)$.
- **Issue:** α has not been defined in the exercise. The exponent should be expressed in terms of δ and λ.
- **Correction / repair:** A standard formulation is: for every

$$0 < \alpha < \min\left\{\gamma, \frac{-\log\delta}{\log\lambda}\right\},$$

the function is C^α; with endpoint conventions one can write the exponent as the corresponding minimum with a minus sign.

C14-M20. Exercise 14.15: the envelope defining V_θ uses the wrong psh class

- **Precise location:** printed p. 386, Exercise 14.15; the exact printed wording or display is reproduced or quoted under **Correction / repair**.
- **Correction / repair:** Replace $\mathrm{PSH}(X, \omega)$ by $\mathrm{PSH}(X, \theta)$.

Typographical and editorial corrections

C14-E01. Proof of Theorem 14.14: the final open set has ambiguous parentheses

- **Precise location:** printed p. 382, proof of Theorem 14.14, last paragraph.
- **Correction / repair:** replace the ambiguous set expression by $X \setminus (E \cup D)$. This is an editorial parenthesis slip and does not affect the argument.

C14-T01. "so called"

- **Precise location:** printed p. 363, Section 14.1; the exact printed wording or display is reproduced or quoted under **Printed text**.
- **Printed text:** so called continuity method.
- **Correction / repair:** so-called continuity method.

C14-T02. Subject-verb agreement

- **Precise location:** printed p. 372, after Theorem 14.9; the exact printed wording or display is reproduced or quoted under **Printed text**.
- **Printed text:** "This results, together with Steps 1 and 2, yields…".
- **Correction / repair:** "This result, together with Steps 1 and 2, yields…" or "These results … yield…".

C14-T03. "allow to control"

- **Precise location:** printed p. 371, discussion before Theorem 14.9; the exact printed wording or display is reproduced or quoted under **Printed text**.
- **Printed text:** the latter allow to control.
- **Correction / repair:** the latter allow one to control.

C14-T04. "Schauder estimtes"

- **Precise location:** printed p. 377, Section 14.3.2; the exact printed wording or display is reproduced or quoted under **Printed text**.
- **Printed text:** Schauder estimtes.
- **Correction / repair:** Schauder estimates.

C14-T05. "boostrapping"

- **Precise location:** printed p. 378, proof of Theorem 14.1; the exact printed wording or display is reproduced or quoted under **Printed text**.
- **Printed text:** A boostrapping argument.
- **Correction / repair:** A bootstrapping argument.

C14-T06. Awkward phrase around Auvray reference

- **Precise location:** printed p. 380, paragraph before Section 14.4.1; the exact printed wording or display is reproduced or quoted under **Printed text**.
- **Printed text:** "as dealt (with a very different method) by Auvray".
- **Correction / repair:** as treated by Auvray, with a very different method or as dealt with by Auvray using a very different method.

C14-T08. Missing space before "for"

- **Precise location:** printed p. 384, Exercise 14.10; the exact printed wording or display is reproduced or quoted under **Printed text**.
- **Printed text:** "(see [GS80]for more details)".
- **Correction / repair:** "(see [GS80] for more details)".

Chapter 15. Canonical metrics

Range checked: printed pp. 389–418.

Mathematical corrections and proof clarifications

C15-M01. Lemma 15.2: the displayed Euler–Lagrange equation for extremal metrics is not the extremal equation

- **Precise location:** printed p. 390, proof of Lemma 15.2; the exact printed wording or display is reproduced or quoted under **Printed text**.
- **Printed text:** after integrating by parts, the text says that ω is extremal iff

$$\frac{\partial^2 \operatorname{Scal}(\omega)}{\partial z_\alpha \partial \bar{z}_\beta} = 0.$$

- **Issue:** The displayed equation is the cscK equation instead.
- **Correction / repair:** Replace it by the extremal-metric condition

$$\bar{\partial}\big(\nabla^{1,0}_\omega \operatorname{Scal}(\omega)\big) = 0,$$

  equivalently, the $(1,0)$-gradient of the scalar curvature is a holomorphic vector field. Vanishing of the full mixed Hessian would force constant scalar curvature and is strictly stronger.

C15-M02. Proposition 15.3: the sign in the entropy-variation computation is wrong, and the final displayed derivative contradicts the definition of the Mabuchi functional

- **Precise location:** printed pp. 391–392, proof of Proposition 15.3; the exact printed wording or display is reproduced or quoted under **Printed text**.
- **Printed text:**
$$\frac{d}{dt}\left\{\frac{1}{V_\alpha}\int_X \log\left(\frac{\omega_t^n}{\omega^n}\right)\omega_t^n\right\} = -\frac{n}{V_\alpha}\int_X \dot{\varphi}_t dd^c \log\left(\frac{\omega_t^n}{\omega^n}\right)\wedge\omega_t^{n-1}.$$
The following page then concludes
$$\frac{d\mathcal{M}_\alpha(\omega_t)}{dt} = \frac{1}{V_\alpha}\int_X\left[n\frac{\mathrm{Ric}(\omega_t)\wedge\omega_t^{n-1}}{\omega_t^n} - \lambda_\alpha\right]\dot{\varphi}_t\omega_t^n.$$
The sign is wrong.
- **Correction / repair:** The entropy derivative begins with the plus sign
$$\frac{n}{V_\alpha}\int_X \dot{\varphi}_t\, dd^c\log\left(\frac{\omega_t^n}{\omega^n}\right)\wedge\omega_t^{n-1}.$$
With the chapter's convention for the Mabuchi functional, the final formula is
$$\frac{d}{dt}\mathcal{M}_\alpha(\omega_t) = -\frac{1}{V_\alpha}\int_X\big(\mathrm{Scal}(\omega_t)-\lambda_\alpha\big)\dot{\varphi}_t\,\omega_t^n.$$

C15-M03. Theorem 15.12: the Moser–Trudinger/properness inequality is false without a normalization or a translation-invariant exhaustion functional

- **Precise location:** printed p. 395, Theorem 15.12; the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** The Ding functional
$$F_1(\varphi) = E(\varphi) + \log\int_X e^{-\varphi}d\mu$$

is invariant under adding constants, whereas
$$E(\varphi+c) = E(\varphi)+c.$$
Thus the printed inequality cannot hold on the full space of Kähler potentials.

- **Correction / repair:** Impose a normalization such as $\sup_X\varphi = 0$, or replace $E(\varphi)$ on the right-hand side by the translation-invariant quantity $E(\varphi) - \sup_X\varphi$ (equivalently, formulate coercivity with a standard J-functional). State the chosen normalization in Theorem 15.12.

C15-M04. Section 15.2.4.2: the definition of F_λ is undefined at $\lambda = 0$

- **Precise location:** printed p. 405, Section 15.2.4.2; the exact printed wording or display is reproduced or quoted under **Correction / repair**.
- **Correction / repair:** Add
$$F_0(\varphi) = E(\varphi) - \int_X \varphi\, d\mu.$$

C15-M05. Proposition 15.16: the Semmes-trick calculation mixes the annulus coordinate with the logarithmic strip coordinate

- **Precise location:** printed p. 399, proof of Proposition 15.16; the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** The annulus coordinate is declared to be $z = e^{t+is}$, but the calculation differentiates as though the complex coordinate were the strip variable $t + is$; the factors z^{-1}, $\bar{z}^{-1}$, and $1/2$ are consequently missing.
- **Correction / repair:** Either use the strip coordinate $\tau = t + is$, for which $d_\tau \Phi = \frac{1}{2}\dot{\varphi}_t(d\tau + d\bar{\tau})$, or keep the annulus coordinate and write

$$d_z\Phi = \frac{\dot{\varphi}_t}{2}\left(\frac{dz}{z} + \frac{d\bar{z}}{\bar{z}}\right).$$

Carry the same factors through the two subsequent wedge computations.

C15-M07. Theorem 15.19 / Remark 15.20: the geodesic statement is not literally a statement inside the smooth space $\mathcal{H}$

- **Precise location:** printed p. 400, Theorem 15.17; and printed pp. 401–402, Theorem 15.19 and Remark 15.20, each place describing the weak geodesic as a path in the smooth space $\mathcal{H}$.
- **Issue:** The geodesic joining two smooth Kähler potentials is generally only weak (at best $C^{1,1}$ in the sense stated here), so its intermediate points need not lie in the smooth Fréchet manifold $\mathcal{H}$.
- **Correction / repair:** place the weak paths in $\mathcal{H}^{1,1}$, with smooth endpoints in $\mathcal{H}$, and refer to the metric completion only when a completion statement is intended. Do not describe the weak geodesic as a smooth path contained in $\mathcal{H}$.

C15-M08. Section 15.2.3.2: the Bergman potential formulas are missing the factor $1/j$

- **Precise location:** printed pp. 402–403, Section 15.2.3.2; the exact printed wording or display is reproduced or quoted under **Printed text**.
- **Printed text:**

$$\varphi_j = \frac{1}{2}\log\sum_{\ell=1}^{s_j}|\sigma_\ell^{(j)}|^2,$$

and

$$\psi_j(h_j) = \frac{1}{2}\log\sum_{\ell=1}^{s_j}|\sigma_\ell^{(h_j)}|^2.$$

- **Correction / repair:** Insert $1/(2j)$ in both Bergman metric formulas.

C15-M09. Theorem 15.22: the metric d_j on $\mathcal{H}_j$ is not defined or normalized

- **Precise location:** printed p. 403, Theorem 15.22; the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** The convergence statement compares the infinite-dimensional Mabuchi distance with a distance d_j on the Bergman symmetric space, but neither the Riemannian metric nor its j-dependent normalization is stated. An unscaled Hilbert–Schmidt metric has the wrong order as $j \to \infty$.
- **Correction / repair:** Define the convention before the theorem. For example, if $H_0, H_1 \in \mathcal{H}_j \simeq GL(s_j, \mathbb{C})/U(s_j)$ and $\lambda_1, \dots, \lambda_{s_j}$ are the eigenvalues of $\log(H_0^{-1}H_1)$, use

$$d_j(H_0, H_1) := \frac{1}{j}\left(\frac{1}{s_j}\sum_{\ell=1}^{s_j}\lambda_\ell^2\right)^{1/2},$$

up to the fixed volume normalization used for the Mabuchi metric. State the convergence through the Hilbert and Fubini–Study comparison maps, using the same convention on both sides.

C15-M10. Section 15.2.2.2: the elliptic curve lattice is written incorrectly

- **Precise location:** printed p. 400, Section 15.2.2.2; the exact printed wording or display is reproduced or quoted under **Printed text**.
- **Printed text:**
$$X = \mathbb{C}/\mathbb{Z}[\tau]$$
Correction / repair: Replace by
$$X = \mathbb{C}/(\mathbb{Z} + \tau\mathbb{Z}).$$

C15-M12. Lemma 15.29 proof: $\iota_V \omega$ is not a "holomorphic $(0,1)$-form", and the potential is not generally real-valued

- **Precise location:** printed p. 408, proof of Lemma 15.29; the exact printed wording or display is reproduced or quoted under **Printed text**.
- **Printed text:** given $V \in H^0(X, T_X)$, the text lets α denote the holomorphic $(0,1)$-form such that
$$\delta_V \omega = \alpha,$$
and then concludes there is $u \in C^\infty(X, \mathbb{R})$ with $\bar\partial u = \alpha$.
- **Correction / repair:** Write $\alpha := \iota_V \omega$. It is a smooth $(0,1)$-form satisfying $\bar\partial\alpha = 0$; it is not a holomorphic form in the usual sense. The $\bar\partial$-lemma gives a generally complex-valued smooth potential u with $\bar\partial u = \alpha$. A real Hamiltonian potential requires an additional reality/Hamiltonian hypothesis (and, depending on conventions, a factor of i). The asserted eigenspace/isomorphism statement should therefore be complexified.

C15-M14. Lemma 15.31: the Futaki/Mabuchi derivative misses the normalization and the displayed proof suppresses a sign change

- **Precise location:** printed pp. 408–409, Definition 15.30 and Lemma 15.31; the exact printed wording or display is reproduced or quoted under **Printed text**.
- **Printed text:** Lemma 15.31 says
$$\left.\frac{d\mathcal{M}(\omega_s)}{ds}\right|_{s=0} = \mathrm{Fut}_\omega(V),$$
while Definition 15.30 sets
$$\mathrm{Fut}_\omega(V) = \int_X V(h_\omega)\omega^n.$$
- **Issue:** First, the Mabuchi functional in this chapter is normalized by V_α^{-1}. With the unnormalized Futaki integral printed in Definition 15.30, the relation should include V_α^{-1}. Second, using the sign convention on printed p. 391,
$$\frac{d\mathcal{M}(\omega_s)}{ds} = -V_\alpha^{-1} \int_X \dot\varphi_s \Delta_{\omega_s} h_s \, \omega_s^n,$$
and the later integration by parts/Lie derivative step is what changes this into $+V_\alpha^{-1} \int_X V(h_s)\omega_s^n$. The printed proof starts directly with the positive, unnormalized integral.

- **Correction / repair:** With Definition 15.30 left unnormalized, state

$$\frac{d}{ds}\mathcal{M}_\alpha(\Phi_s^*\omega)\Big|_{s=0} = V_\alpha^{-1}\,\mathrm{Fut}_\omega(V).$$

  In the proof, first insert the defining negative Mabuchi variation and only then use integration by parts together with $L_V\omega_s = dd^c\dot\varphi_s$ to obtain the positive Futaki integral.

C15-M16. Example 15.40 and Exercise 15.13: the family of conics is not a test configuration for $(\mathbb{P}^2, \mathcal{O}(1))$

- **Precise location:** printed pp. 412 and 416, Example 15.40 and Exercise 15.13; the exact printed wording or display is reproduced or quoted under **Printed text**.
- **Printed text:** the family

$$\{[x:y:z]\in\mathbb{P}^2 \mid xy = tz^2\}_{t\in\mathbb{C}}$$

  defines a test configuration for $(X, L) = (\mathbb{P}^2, \mathcal{O}(1))$ of exponent 2.
- **Issue:** The fibers of this family are conics in $\mathbb{P}^2$, hence one-dimensional. They are not isomorphic to $\mathbb{P}^2$, so this cannot be a test configuration for $(\mathbb{P}^2, \mathcal{O}(1))$.
- **Correction / repair:** The family is a degeneration of a smooth conic, hence of $\mathbb{P}^1$, to the union of two lines. Describe it as a test configuration for $(\mathbb{P}^1, \mathcal{O}_{\mathbb{P}^1}(2))$ of exponent 1, or equivalently for $(\mathbb{P}^1, \mathcal{O}_{\mathbb{P}^1}(1))$ of exponent 2, according to the chosen polarization convention.

C15-M17. Theorem 15.43: K-stable should be replaced by K-polystable unless automorphisms are excluded

- **Precise location:** printed pp. 412–413, Definition 15.41 and Theorem 15.43; the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** A Kähler–Einstein Fano manifold with non-discrete automorphism group has product test configurations of zero Donaldson–Futaki invariant, so it need not be K-stable in the strict sense of Definition 15.41.
- **Correction / repair:** State Theorem 15.43 with K-*polystable*. The strict K-stable formulation is valid after imposing the discreteness/no-nonzero-holomorphic-vector-field hypothesis used in Conjecture 15.42.

C15-M19. Section 15.4.3: the statement that the limiting pair (V, Δ) is log smooth is too strong

- **Precise location:** printed pp. 413–414, sketch of the Chen–Donaldson–Sun method, Section 15.4.3; the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** The Gromov–Hausdorff limit is generally a singular $\mathbb{Q}$-Fano variety and the limiting divisor need not have simple normal crossings on it. Thus the pair is not generally log smooth.
- **Correction / repair:** replace the phrase by the precise statement that

$$\left(V, \frac{1-T_{\max}}{m}\Delta\right)$$

  is the relevant klt/log-Fano pair. The limit V may be singular; log smoothness is available only after passing to a log resolution.

C15-M20. Exercise 15.6: the definition of φ_j is not well-defined on projective space as printed

- **Precise location:** printed p. 415, Exercise 15.6; the exact printed wording or display is reproduced or quoted under **Printed text**.
- **Printed text:**

$$\varphi_j[z] = (1-\varepsilon_j)\varphi_0[z] + \varepsilon \max(\log|z_1|, \log|z_0| - C_j) \in \mathrm{PSH}(\mathbb{P}^1, \omega_{FS}).$$

- **Issue:** The term

$$\max(\log|z_1|, \log|z_0| - C_j)$$

is not homogeneous.

- **Correction / repair:** Subtract the Fubini–Study weight so the expression is invariant under homogeneous rescaling, and use the same coefficient ε_j in both terms. For example, with $\omega_{FS} = dd^c \frac{1}{2}\log(|z_0|^2 + |z_1|^2)$, replace the last term by

$$\varepsilon_j \left[\max\{\log|z_1|, \log|z_0| - C_j\} - \tfrac{1}{2}\log(|z_0|^2 + |z_1|^2)\right].$$

C15-N01. Exercise 15.7: the variety in the conclusion has the wrong symbol

- **Type:** notation error.
- **Precise location:** printed p. 415, Exercise 15.7, final instruction "Show that X is not Fano."
- **Correction / repair:** replace X by X_r, the blow-up introduced in the exercise.

Typographical and editorial corrections

C15-E01. Definition 15.30: a representative form is called a class

- **Precise location:** printed p. 408, Definition 15.30, phrase "For any Kähler class $\omega \in \mathcal{K}_\alpha$."
- **Correction / repair:** replace "Kähler class" by "Kähler form." The class is α; ω is its representative.

C15-T01. Missing article in "as possible generalization"

- **Precise location:** printed p. 390; the exact printed wording or display is reproduced or quoted under **Printed text**.
- **Printed text:** "as possible generalization".
- **Correction / repair:** "as possible generalizations" or "as a possible generalization".

C15-T02. "know" should be "now"

- **Precise location:** printed p. 392; the exact printed wording or display is reproduced or quoted under **Printed text**.
- **Printed text:** "as we know explain".
- **Correction / repair:** "as we now explain".

C15-T04. "homogenous" should be "homogeneous"

- **Precise location:** printed p. 399; the exact printed wording or display is reproduced or quoted under **Printed text**.
- **Printed text:** "homogenous complex Monge–Ampère equation".
- **Correction / repair:** "homogeneous complex Monge–Ampère equation".

C15-T05. Subject–verb agreement

- **Precise location:** printed p. 400; the exact printed wording or display is reproduced or quoted under **Printed text**.
- **Printed text:** “there exists geodesics”.
- **Correction / repair:** “there exist geodesics”.

C15-T06. “adressed” should be “addressed”

- **Precise location:** printed p. 402; the exact printed wording or display is reproduced or quoted under **Printed text**.
- **Printed text:** “adressed”.
- **Correction / repair:** “addressed”.

C15-T07. “many information” should be “much information”

- **Precise location:** printed p. 403; the exact printed wording or display is reproduced or quoted under **Printed text**.
- **Printed text:** “many additional information”.
- **Correction / repair:** “much additional information”.

C15-T08. Comma splice

- **Precise location:** printed p. 406; the exact printed wording or display is reproduced or quoted under **Printed text**.
- **Printed text:** comma splice after “there are 105 families!”.
- **Correction / repair:** Split the sentence or replace the comma by a semicolon/period.

C15-T09. Duplicated “from”

- **Precise location:** printed p. 410; the exact printed wording or display is reproduced or quoted under **Printed text**.
- **Printed text:** “It follows from from Chapter 11”.
- **Correction / repair:** “It follows from Chapter 11”.

C15-T10. Space before period

- **Precise location:** printed p. 411; the exact printed wording or display is reproduced or quoted under **Printed text**.
- **Printed text:** “vanishes .”.
- **Correction / repair:** “vanishes.”.

C15-T12. Missing linear-system bars

- **Precise location:** printed p. 413; the exact printed wording or display is reproduced or quoted under **Printed text**.
- **Printed text:** “if there exists one such in $-K_X$”.
- **Correction / repair:** “if there exists such a divisor in $|-K_X|$”.

C15-T13. Repeated "previous"

- **Precise location:** printed p. 414; the exact printed wording or display is reproduced or quoted under **Printed text**.
- **Printed text:** "A previous important result along these lines had previously been achieved".
- **Correction / repair:** Remove one occurrence of "previous".

C15-T14. "in details" should be "in detail"

- **Precise location:** printed p. 414; the exact printed wording or display is reproduced or quoted under **Printed text**.
- **Printed text:** "explain it in details".
- **Correction / repair:** "explain it in detail".

Chapter 16. Singularities and the Minimal Model Program

Range checked: printed pp. 419–450.

Mathematical corrections and proof clarifications

C16-M01. Definition 16.3: semi-ampleness is conflated with global generation and finite generation of the section ring

- **Precise location:** printed p. 420, Definition 16.3 and the paragraph immediately after it; the exact printed wording or display is reproduced or quoted under **Printed text**.
- **Printed text:** after defining semi-ampleness by the existence of k, a morphism $f : X \to Y$, and an ample line bundle A with $L^k = f^*A$, the text says that one also says that L^k is globally generated, "i.e." the section ring

$$R(X, L) = \bigoplus_{m \geq 0} H^0(X, L^{mk})$$

is finitely generated, and then says the two notions are equivalent.

- **Issue:** Global generation of one power and finite generation of the section ring are different statements; the displayed "i.e." identifies them.
- **Correction / repair:** Say that L is semi-ample iff some positive power L^k is globally generated. This is equivalent to the displayed factorization $L^k \simeq f^*A$ through a morphism and an ample line bundle after replacing the power if necessary. Semi-ampleness implies finite generation of the section ring, but finite generation alone is not equivalent: one must also exclude a stable base locus (equivalently, require an appropriate Veronese power to be generated in degree one and basepoint free). Thus delete the printed "i.e." before the section-ring statement.

C16-M02. Definition 16.4: surjectivity is missing

- **Precise location:** printed p. 420, paragraph after Definition 16.4; the exact printed wording or display is reproduced or quoted under **Printed text**.
- **Printed text:** for a proper holomorphic map $f : X \to Y$,

$$L \text{ is nef} \quad \Longleftrightarrow \quad f^*L \text{ is nef.}$$

- **Issue:** The implication f^*L nef $\Rightarrow L$ nef fails if f is not surjective. For example, if f is constant, f^*L is trivial and hence nef for every L, whether or not L is nef on Y.
- **Correction / repair:** Add that f is surjective. For a surjective proper morphism of projective varieties, L is nef iff f^*L is nef; without surjectivity only the forward implication is valid.

C16-M03. After Definition 16.21: local factoriality does not supply Cohen–Macaulayness

- **Precise location:** printed p. 429, first sentence after Definition 16.21.
- **Printed text:** *Thus a normal variety which is locally factorial (resp. $\mathbb{Q}$-factorial) is Gorenstein (resp. $\mathbb{Q}$-Gorenstein).*
- **Issue:** local factoriality makes the canonical Weil divisor Cartier, hence makes the variety 1-Gorenstein in Definition 16.18, but it does not by itself imply the Cohen–Macaulay condition that the same definition requires for *Gorenstein*.
- **Correction / repair:** replace the first implication by: *a locally factorial normal variety is* 1*-Gorenstein, and is Gorenstein if it is also Cohen–Macaulay.* The parenthetical implication $\mathbb{Q}$-factorial $\Rightarrow$ $\mathbb{Q}$-Gorenstein is valid under Definition 16.19 and may be retained.

C16-M06. Definition 16.22: the factorization of the minimal resolution has the composition order reversed

- **Precise location:** printed p. 429, paragraph after Definition 16.22; the exact printed wording or display is reproduced or quoted under **Printed text**.
- **Printed text:** if $\pi : X \to V$ is minimal, then any other resolution $\pi' : X' \to V$ factorizes through $f : X' \to X$ such that
$$\pi' = f \circ \pi.$$
- **Correction / repair:** Replace $\pi' = f \circ \pi$ by $\pi' = \pi \circ f$.

C16-M07. Theorem 16.24: Zariski connectedness is false for arbitrary proper morphisms

- **Precise location:** printed p. 429, Theorem 16.24; the exact printed wording or display is reproduced or quoted under **Printed text**.
- **Printed text:** if $\pi : X \to V$ is a proper morphism between normal complex analytic spaces, then all fibers $\pi^{-1}(p)$ are connected.
- **Issue:** This is false as stated.
- **Correction / repair:** State for a proper bimeromorphic morphism or add the hypothesis $\pi_*\mathcal{O}_X = \mathcal{O}_V$.

C16-M09. Definition 16.27: the pole-order bound for log-terminal singularities is off by one

- **Precise location:** printed p. 430, Definition 16.27; the exact printed wording or display is reproduced or quoted under **Printed text**.
- **Printed text:** for a local generator α of $\omega_V^{[N]}$, the pole of $\pi^*\alpha$ along any exceptional component has order $< N - 1$.
- **Correction / repair:** Replace $< N - 1$ by $< N$.

C16-M10. Definitions 16.29 and 16.32: the definitions of klt/log canonical pairs miss the coefficients of the boundary itself

- **Precise location:** printed pp. 431–432, Definitions 16.29 and 16.32; the exact printed wording or display is reproduced or quoted under **Printed text**.
- **Printed text:** The discrepancy sums run only over exceptional divisors, while coefficient bounds for the strict transform Δ' are not stated.
- **Issue:** Klt and log-canonical conditions also constrain divisors already present on V. For a smooth simple-normal-crossing pair, for example, klt requires every boundary coefficient to be < 1, whereas lc requires it to be ≤ 1.

- **Correction / repair:** Either quantify over every prime divisor over V, including non-exceptional divisors, or supplement the printed discrepancy inequalities with: all coefficients of Δ are < 1 for klt and ≤ 1 for log canonical.

C16-M11. Proposition 16.14 proof: dd^c is missing in the pullback formula

- **Precise location:** printed p. 425, proof of Proposition 16.14; the exact printed wording or display is reproduced or quoted under **Printed text**.
- **Printed text:** after $T = [D] + dd^c u$, the text writes
$$q^*T = [z_2 = 0] + u \circ q.$$
- **Issue:** The right-hand side mixes a current of integration with a function.
- **Correction / repair:** Restore the missing dd^c:
$$q^*T = [z_2 = 0] + dd^c(u \circ q).$$

C16-M12. Example 16.39: the equation of $\mathbb{C}^2/\pm 1$ in $\mathbb{C}^3$ is wrong

- **Precise location:** printed p. 434, Example 16.39; the exact printed wording or display is reproduced or quoted under **Printed text**.
- **Printed text:**
$$V \simeq \{(u, v, w) \in \mathbb{C}^3 \mid u = x^2, v = y^2, w = xy\}.$$
- **Issue:** This is not an equation in the coordinates (u, v, w) on $\mathbb{C}^3$; it still uses the covering coordinates (x, y).
- **Correction / repair:** Replace the displayed set by
$$V \simeq \{(u, v, w) \in \mathbb{C}^3 \mid uv = w^2\}, \qquad u = x^2, \quad v = y^2, \quad w = xy.$$

C16-M13a. Proposition 16.42: the dominating resolution notation has wrong spaces

- **Precise location:** printed p. 436, Proposition 16.42, sentence beginning *Moreover, if* $\bar{\pi} : \bar{X} \to V$ *is a resolution dominating* π.
- **Correction / repair:** write $\bar{\pi} = \pi \circ \psi$ with $\psi : \bar{X} \to X$.

C16-M13b. Lemma 16.43: the bounded potential is assigned to the wrong space

- **Precise location:** printed p. 436, Lemma 16.43, hypothesis after $\theta_1 = \theta_2 + dd^c\varphi$.
- **Correction / repair:** replace $\varphi \in L^\infty(X)$ by $\varphi \in L^\infty(V)$.

C16-M16. Theorem 16.62: the normalization contains an impossible factor $(-1)^n$

- **Precise location:** printed p. 444, Theorem 16.62; the exact printed wording or display is reproduced or quoted under **Printed text**.
- **Printed text:**
$$\int_V \theta^n = C \int_V (-1)^n \nu_\alpha.$$
- **Issue:** Both θ^n and the canonical measure ν_α are positive. In odd complex dimension the displayed normalization would equate a positive integral with a negative one.
- **Correction / repair:** Delete $(-1)^n$ and choose $C > 0$ by
$$\int_V \theta^n = C \int_V \nu_\alpha.$$

C16-M17. Lemma 16.72: X_0 is used without definition

- **Precise location:** printed p. 447, Lemma 16.72; the exact printed wording or display is reproduced or quoted under **Issue**.
- **Issue:** The notation X_0 occurs in the lemma before it has been defined.
- **Correction / repair:** Define
$$X_0 = X_{\mathrm{reg}} \setminus \mathrm{Supp}(D)$$
immediately before Lemma 16.72, or replace X_0 by this set in the statement.

C16-N02. Corollary 16.52: the exceptional divisor is taken under the wrong map

- **Type:** type-invalid map expression.
- **Precise location:** printed p. 440, proof of Corollary 16.52, after $b : X' \to \widetilde{X}$ is defined as the blow-up of $p \in \widetilde{X}$ and $\pi' = \pi \circ b : X' \to X$; PDF p. 465.
- **Printed text / issue:** $E = \pi'^{-1}(p)$ is meaningless because p lies in $\widetilde{X}$, not in the target X of π'.
- **Correction / repair:** set $E = b^{-1}(p)$. Then E is the exceptional prime divisor and $\pi'(E) = x$, as required for the application of Theorem 16.51.

C16-N03. Lemma 16.53: the proof uses an undefined divisor D

- **Type:** formula symbol error.
- **Precise location:** printed p. 441, proof of Lemma 16.53, display beginning $(-D|_{E_i}) \cdot (\omega|_{E_i})^{n-2}$; PDF p. 466.
- **Printed text / issue:** the lemma defines $B = \sum_j b_j E_j$, not D, and the hypothesis concerns $-B|_{E_i}$.
- **Correction / repair:** replace D by B in that display.

Typographical and editorial corrections

C16-N01. After Definition 16.21: missing noun after "Q-factorial"

- **Precise location:** printed p. 429, paragraph immediately after Definition 16.21, phrase "with V an algebraic Q-factorial."
- **Correction / repair:** write "with V a Q-factorial algebraic variety."

C16-N04. Exercise 16.2: semi-ampleness is undefined for the stated real class

- **Precise location:** printed p. 449, Exercise 16.2, request for a real class $\alpha \in H^{1,1}(X, \mathbb{R})$ that is "semi-positive, but not semi-ample"; PDF p. 474.
- **Issue:** Section 16.1 defines semi-ampleness for line bundles, not arbitrary real cohomology classes.
- **Correction / repair:** either define the intended extension of semi-ampleness to real classes, or ask for a line bundle L such that $c_1(L)$ is semipositive but L is not semiample.

C16-E01. Four local symbol slips reclassified as editorial corrections

- printed p. 439, Lemma 16.48, conclusion: $(X, \Delta) \longrightarrow (V, \Delta)$.
- printed p. 444, Definition 16.61, phrase "some multiple N' of $\mathrm{index}(X)$": use $\mathrm{index}(V)$.
- printed p. 446, Definition 16.66, first sentence defining a $\mathbb{Q}$–Calabi–Yau pair: use $\mathrm{index}(V, \Delta)$, not $\mathrm{index}(X, \Delta)$.
- printed p. 448, proof of Proposition 16.73, local equality: replace $V \cap \Omega$ by $U \cap \Omega$, since U is the local open set just fixed.

C16-T01. printed p. 420: if an only if

- **Precise location:** printed p. 420; the exact printed wording or display is reproduced or quoted under **Printed text**.
- **Printed text:** "C on a projective surface X is nef if an only if $C^2 \geq 0$".
- **Correction / repair:** if and only if.

C16-T02. printed p. 422: to being psef

- **Precise location:** printed p. 422; the exact printed wording or display is reproduced or quoted under **Printed text**.
- **Printed text:** "This is stronger than asking for K_V to being psef".
- **Correction / repair:** "to be psef".

C16-T03. printed p. 423: missing closing parenthesis

- **Precise location:** printed p. 423; the exact printed wording or display is reproduced or quoted under **Printed text**.
- **Printed text:** "see the corresponding discussion in Section 16.2."
- **Correction / repair:** "see the corresponding discussion in Section 16.2)."

C16-T04. printed p. 427: his fractions field

- **Precise location:** printed p. 427; the exact printed wording or display is reproduced or quoted under **Printed text**.
- **Printed text:** "integrally closed in his fractions field".
- **Correction / repair:** "integrally closed in its fraction field".

C16-T05. printed pp. 427–428: an holomorphic

- **Precise location:** printed pp. 427–428; the exact printed wording or display is reproduced or quoted under **Printed text**.
- **Printed text:** an holomorphic n-form; an holomorphic pluricanonical form.
- **Correction / repair:** a holomorphic n-form; a holomorphic pluricanonical form.

C16-T06. printed p. 429: midly singular

- **Precise location:** printed p. 429; the exact printed wording or display is reproduced or quoted under **Printed text**.
- **Printed text:** midly singular.
- **Correction / repair:** mildly singular.

C16-T07. printed p. 434: equivalence classes

- **Precise location:** printed p. 434; the exact printed wording or display is reproduced or quoted under **Printed text**.
- **Printed text:** A positive current on V is an equivalence classes of plurisubharmonic potentials.
- **Correction / repair:** an equivalence class.

C16-T08. printed p. 434: duplicated for all i

- **Precise location:** printed p. 434; the exact printed wording or display is reproduced or quoted under **Printed text**.
- **Printed text:** "$\varphi_i + \varphi$ is psh on U_i for all i" after the sentence already says "for all i".
- **Correction / repair:** delete the second for all i.

C16-T09. printed p. 434: tranform

- **Precise location:** printed p. 434; the exact printed wording or display is reproduced or quoted under **Printed text**.
- **Printed text:** take the strict tranform of V.
- **Correction / repair:** strict transform.

C16-T10. printed p. 434: wrong ambient letter in $H^1(X, \mathcal{PH}_X)$

- **Precise location:** printed p. 434; the exact printed wording or display is reproduced or quoted under **Printed text**.
- **Printed text:** after discussing a normal space V, the text says "A class in $H^1(X, \mathcal{PH}_X)$".
- **Correction / repair:** $H^1(V, \mathcal{PH}_V)$.

C16-T11. printed p. 435: missing "be" in Proposition 16.41

- **Precise location:** printed p. 435, Proposition 16.41; the exact printed wording or display is reproduced or quoted under **Printed text**.
- **Printed text:** Let V a compact normal complex analytic variety and let L a holomorphic line bundle on V.
- **Correction / repair:** Let V be ... and let L be ....

C16-T12. printed p. 437: wrong space name in Lemma 16.45 proof

- **Precise location:** printed p. 437, proof of Lemma 16.45; the exact printed wording or display is reproduced or quoted under **Printed text**.
- **Printed text:** "f_1 belongs actually to $L^p(X, d\lambda)$ for some $p > 1$ when X is log terminal."
- **Correction / repair:** "when V is log terminal".

C16-T13. printed p. 438: kltpair

- **Precise location:** printed p. 438; the exact printed wording or display is reproduced or quoted under **Printed text**.
- **Printed text:** "Let (V, Δ) be a kltpair."
- **Correction / repair:** klt pair.

C16-T14. printed p. 438: staightforward

- **Precise location:** printed p. 438; the exact printed wording or display is reproduced or quoted under **Printed text**.
- **Printed text:** the following staightforward extension.
- **Correction / repair:** straightforward.

C16-T15. printed p. 439: This is the contents

- **Precise location:** printed p. 439; the exact printed wording or display is reproduced or quoted under **Printed text**.
- **Printed text:** This is the contents of this section.
- **Correction / repair:** This is the content of this section.

C16-T16. printed p. 443: missing "be" in Theorem 16.58

- **Precise location:** printed p. 443, Theorem 16.58; the exact printed wording or display is reproduced or quoted under **Printed text**.
- **Printed text:** "Let h^N a smooth Hermitian metric…".
- **Correction / repair:** "Let h^N be a smooth Hermitian metric…".

C16-T17. printed p. 443: malformed Chapter 14 reference

- **Precise location:** printed p. 443; the exact printed wording or display is reproduced or quoted under **Printed text**.
- **Printed text:** "Moreover, from Chapter it follows 14 that ψ is smooth…".
- **Correction / repair:** "Moreover, it follows from Chapter 14 that ψ is smooth…".

C16-T18. printed p. 444: needs not

- **Precise location:** printed p. 444; the exact printed wording or display is reproduced or quoted under **Printed text**.
- **Printed text:** It needs not be a singular KE metric…
- **Correction / repair:** It need not be… or It does not need to be….

C16-T19. printed p. 445: midly singular

- **Precise location:** printed p. 445; the exact printed wording or display is reproduced or quoted under **Printed text**.
- **Printed text:** midly singular hypersurfaces.
- **Correction / repair:** mildly singular hypersurfaces.

C16-T20. printed p. 447: implicitely

- **Precise location:** printed p. 447; the exact printed wording or display is reproduced or quoted under **Printed text**.
- **Printed text:** "Writing $\mathrm{Ric}(\omega)$ on X^{reg} implicitely means…".
- **Correction / repair:** implicitly.

C16-T21. printed p. 448: subject-verb agreement

- **Precise location:** printed p. 448; the exact printed wording or display is reproduced or quoted under **Printed text**.
- **Printed text:** There has been important recent works…
- **Correction / repair:** There have been important recent works…

Back matter / index notes

- **B-N01. Precise location:** bibliography, printed p. 458, entry [Don99], journal/series field "AXS Translations Series 2(196)." **Correction / repair:** "AMS Translations, Series 2, vol. 196."
- **B-N02. Precise location:** bibliography, printed p. 461, entry [Kol98], which gives author, journal, volume, year, and pages but omits the article title. **Correction / repair:** insert "The complex Monge–Ampère equation" before *Acta Math.* 180 (1998), no. 1, 69–117.
- **B-M01. Precise location:** Index, printed p. 470, left column, entry *determinent bundle*. **Correction / repair:** *determinent bundle* $\longrightarrow$ *determinant bundle*.
- **B-M02. Precise location:** Index, printed p. 470, right column, entry *Itaka dimension*. **Correction / repair:** *Itaka dimension* $\longrightarrow$ *Iitaka dimension*.